When we came across the fractal geometry – and results nobody likes

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This *philosophical short note* tries to deepen several ideas/results in/of *our* heuristic and tentative work on the subject since its accidental start in 2004. Long time before, one of us (F.J.) had gotten three key stimuli: i) the late A. Einstein's (1954, been aware of since 1981, Michele Besso-Correspondence) statement that he considers it quite possible that physics cannot be based on the field concept, i.e. on continuous structures…, ii) fields in J.A. Wheeler's diction (1973, in Gravitation, C.W. Misner, K.S. Thorne & J.A. Wheeler) as modes of a substrate… that is not and will not be revealed by reasoning from the top down, only from the bottom up…, and iii) J.D. Bekenstein's (1972, PhD thesis) black hole entropy formula. Well beyond one's capabilities then, that's why this maybe indicated the likely existence of another, much more profound "thermodynamics" than the known one.

Still adhering to the neo-Einsteinian conviction, the master's late time in Princeton (when widely been considered as an old fool there, in his own diction) is anything but thought of as a wasted one. A failure of the field concept, alternatively final description of nature by some very general statistics since has got much potential of revolutionizing fundamental physics. Unfortunately *we* haven't (me being experimentalist, sharing M. Faraday's lousy performance in mathematics but not his technical skills, and nephew Werner being an IT-specialist still studying maths). I got "trapped" by fractal geometry after having bought the inspiring book "The Beauty of Fractals" (by H.-O. Peitgen & P.H. Richter, 1986) in 1987. The mere beauty of (artfully coloured) Mandelbrot set potential photos there immediately made me ask for computational help from O. Lippitsch (a colleague and "owner" of one of our then employer's first efficient personal computers, with close contact to the utility's mainframe people). Mandelbrot set's potential, details from the so-called seahorse valley etc. as black-and-white photos soon were generated, the machine stably run over night. The whole effort just for the sake of beauty then! My first contact with erratic dynamics was in 1983 when working on logistic substitution then (asking ourselves whether energy markets were chaotic, F.J. Culetto & A. Hofstaetter, unpublished), following the footsteps of C. Marchetti's (1979) pioneering work based on results by J.C. Fisher & R.H. Pry (GE, 1970). Common experience at work, observation of what was going on in energy policy (and utilities' committees) did enhance one's impression that chaos be the normal case, and order is the exception. And two decades later eased the step towards iterative maps.

Late in 2004 then, pretty tired and looking for a detail concerning radiative corrections in Landau-Lifschitz' ('69/75) textbook vol. IVb /relativistic quantum theory, I did "perceive" an approximate expression there: for the Planck mass - electron mass ratio (obviously read between the lines, one of the numbers still unknown in 1975) being approx. twice an exp-function of a square root, therein a term with Euler's γ times some other constants, minus (or +?) a log(?, or 1/log) - term of a product of δ^2 (latter δ for sure Feigenbaum's universal number, also seen in a 2D-phase-transition context before), something else and the fine-structure constant α . The detail looked for eventually not found there, fallen asleep in situ then, late after midnight. The days after (my memory did still serve me unusually well then) belonged to α-numerology. If this sort of procedure was going to make sense at all, then with dimensionless quantities or ratios only. And the fine-structure constant's pre-factor "perceived", didn't it look like the inverse of a Gaussian-distribution-density squared exp-function's one, i.e. $2\pi\sigma^2$? As this expression, when $\sigma \to \delta$, comes quite close to 137, we took $(2\pi\delta^2)^{-1}$ for α_0 and further tried to reproduce $\alpha_{CODATA\ 2002}$ via multiplication of α_0 by $\exp(-X^2)$, anticipating $\varphi(\sigma X;\ \sigma=\delta)^2$'s formula. For the just small correction needed, a pretty large Z_c^2 term ($X = Z_c^{-1}$) had to be found. R.P. Feynman's old question, whether the [electrodynamics] coupling's number is related to pi [π] or perhaps to the base [e] of natural logarithms (and nobody knows) had already been "trivially answered" by our ansatz. But his thoughts' known intricacy suggested adding of a subtle, e↔π dual dependence of $\sim e^\pi \pi^e$ like shape (commuting operators and existence of the limits taken for granted). A week or so was spent in search of such fitting big correction term, which finally turned out to be $Z_c^2 = \gamma e^{\pi+1}\pi^{e+1}$, γ being the Euler-(Mascheroni) constant already mentioned above.

Some more time was needed to realize the connection of the "perceived" unclear M_P/m_e-formula (at moderate confidence in it) and the fine-structure constant approximation's Z_c^2. As soon as was found out that $\exp(\gamma^{1/2} e^{\pi/2+1/2}\pi^{e/2+1/2})$ is of Planck mass - electron pair rest-mass ratio's order of magnitude, a corresponding tentative/heuristic fit was planned and later done at pretty good precision (see Eq.2 of our sciencephilosophy.pdf file). What remained to be completed was the search for fine-structure constant approximation's still missing small correction term(s), the exp( )-function's final argument assumed to be of $-1/\{\gamma(e^{\pi+1}\pi^{e+1} - Z_{small}^2)\}$ shape. The idea, that one could have caught twice the leading term of a hyperbolic sine (or -cosine, such often found in the 2D phase transition context) but missed twice its small term just gave too small a correction. Feigenbaum universal number's appearance, if not just accidental, told that one had to deal with period doubling and bifurcation roots /root-distances were going to play some role. As nonlinear complex dynamics probably was behind it all, a Myrberg sequence of binary bifurcations like case (this time its complex plane extension, the main sequence of binary bifurcations on the real c-axis of the Mandelbrot set) seemed to be worth studying. So external angles ("field lines' " direction in the infinite distance limit, the concept by J.H. Hubbard & A. Douady), especially the one tied to the infinite bifurcation limit (its specific angle being the Thue-Morse /parity constant P = 0.412454…) got involved. And our trial-and-error method fit result, got for the small correction still missing, was $Z_{small}^2 = \pi P/2$. The later interpretation of the term not yet found. But what could be shown was the M_P/m_e approximation's shape in case of good fit by rewriting this in terms of the fine-structure constant approximation: $M_P/m_e \approx 2A\exp((\gamma\pi P/2 - 1/\ln(2\pi\delta^2\alpha))^{1/2})$, reproducing all of the details strangely perceived as told above and even more. Thanks to such pretty exotic epistemological route, indeed. After completion of the fit procedures /approximation formulae looked for, i.e. find of the shape to the numerical value of the pre-factor A, fine-structure constant's approximation formula was rewritten in terms of the M_P/m_e approximation. And the large correction term ended up as $Z_c^2 = \ln(CM_P^2/m_e^2)^2/4$, C = 0.160 987…(see our file sciencephilosophy4.pdf). The $\sim\ln(\ )^2$ - sum term reminded me of the there mutually compensating ones in charge renormalization to fourth order, seen along Bjorken/Drell textbook's reproduction of Jost/ Luttinger's 1950 result. Was summing up the corrections for all orders in α indeed going to come up with the sort of term(s) falling out on the way? A strange situation maybe true, a likely explanation about the fact that charge renormalization fails in the high order limit? Sometime later, we noticed that Z_{small}^2 can be rewritten in terms of the Mandelbrot set's real c-axis values ($\in\partial M$) and accessory external angles ξ(c), $Z_{small}^2 = \Gamma(\xi(-2))^2\xi(-2)\xi(c_D)$, containing the square of the geometric mean of the external angles accessory to – 2, the Mandelbrot set's left end, and to c_D , the Myrberg-Feigenbaum point, respectively. Some sort of a link between the *external angles* $\xi(c_k)$, $k=2^n$, $n=0,1,2,\ldots$ accessory to bifurcation roots of Mandelbrot set's main series and *relative electric charge* of quantum particles (at least for the series' starting point and the first bifurcation) was going to manifest itself. In this context, one has to recall the fact that the whole of the argumentation using Mandelbrot set features depends on a mathematical situation dealing with (nonlinear) iterative maps. Illustrated by the iterative $z_{n+1}= z_n^2+ c$ map (the mathematical situation where the Mandelbrot set is going back to, there fixing $z_0=0$ and varying the complex c-parameter) one immediately notices, that both variable and parameter cannot be dimensional, e.g. [m/s], [kg] or [J] etc. So only dimensionless quantities & ratios count and are independent of the actual frame. And if nature indeed used Mandelbrot's set as (a) control space, and M's universality had been capable of most of the physical forces' unification (=DHM-unification, after Douady/Hubbard/Mandelbrot, if we were to name such case), at least microphysics would likely look different from what it does in current theory. It'll take some time to get used to the fact that everything depend on a single (or just a few) parameter(s), at ultra-high temperature already being (a) complex tensor quantity/ies: from our view of the minimum geometry, $c_{klm..}(T)$'s (themselves fluctuating) elements at least being the real and imaginary part of two c-vectors (phase functions in parameter planes) corresponding to the two (orthogonal) complex z-variable planes needed for a (real) 2D-surface with highly nonlinear complex dynamics, further the image from the phase-mapping of a small (spatial) extra-dimension likely used for $z_{1,2}$-fluctuations' reduction purposes. Unification doesn't seem to be for free. And if the c-parameter tensor by its contracted /vector /scalar version's "modulus"/suitable projection were allowed to directly represent the dimensionless ratio $v(c)/c_0$, where $v(c)$ if $>c_0$ is a value from the photons' normally forbidden velocity range and c_0 the regular speed of light, Mandelbrot set's universality – guaranteed for c not exceeding 4 in modulus – would allow for photon group-velocities up to four times the normal speed of light. Such nowadays in all probability are observable within some exotic context (1D-map nonlinear dynamics?) only, more likely in an early stage of the universe.

J.D. Bekenstein had been referred to in the beginning. And one's since feeling that there might be another "thermodynamics" at work, more profound than the known one. As far as our tentative studies are concerned, "± everything" could be perceived "as an invariant outcome of low-D, nonlinear complex (chaotic quantum?)dynamics of underlying iterative maps". And latter (in a maybe naïve manner) taken in their simplest form: quadratic ones pretty like Einstein's then restriction to at most second order differential equations, about which S. Weinberg (1979) said that "this certainly worked well, but it leaves us wondering why nature should care about whether the field equations are of second or higher order." Within our framework, double iteration using the $R(z^{(1,2)}) = ((z^{(1,2)}-2)/z^{(1,2)})^2$ mapping, i.e. $z^{(1,2)}_{n+1}= R(z^{(1,2)}_n)$ by its Julia sets $J_R = \mathbb{C} \cup \{ \infty \}$ (see our file sciencephilosophy6.pdf and lit. cit.) generates the surface dictating /carrying the real-space dynamics. Analogous, but by elements from two parameter planes, target-sheets for "phase mapping" are provided. Some iterative, area-preserving 2D-map's presence (→Feigenbaum's δ_{2D}) is needed too. And the iterative $z \rightarrow z^2+ c$ map finally expresses Mandelbrot set's combinatorial features via adjoined Julia set $J_c(z)$'s dynamics. Conclusions from the conjecture have been drawn by us. And here is another, fatal one: there won't be no algebraic "formula of everything", owing to the lack of $J_c(z)$'s algebraic formulation, and if so, only a phenomenological one.