Cosmological finds from Mandelbrot set's universality?

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Abstract
This *philosophical contribution* tries to trace remnants of highly nonlinear 1-2D dynamics in (our) universe's very early stage, attributable to Mandelbrot set M's action as a control space (if any). M's universality should possibly have put constraints on matter↔geometry interplay, and space-time came out ± discrete according to the underlying iterative maps.

Out of nothingness, except for maths' concepts & energy on tick?
Mandelbrot set (1) M's universality (mathematicians prefer structural stability (2)) had the potential of influencing all structures /things existing, from their early stages on, if M been used as (a) control space by nature. All of M's combinatorial features are preserved if the holomorphic function F(z) iterated (properly rescaled, 0 trapped in a bounded region) just differs from $(z^2 + c)$ by <1 for all z-values, and parameter c does not exceed 4 in modulus (2). If iteration theory had been to drive evolution, the $R(z) = ((z – 2)/z)^2$ mapping did add its Julia set, $J_R = \mathbb{C} \cup \{ \infty \}$ (3, 4), the closed z-plane, so generating an invariant 2D space capable of giving birth to things by the holographic principle[(5)], system's dynamics entirely dictated by its surface. Objects therein ab initio came into being mediated by Julia sets J_c belonging to the iterative $z \rightarrow z^2 + c$ map (the simplest and the only one where everything depends on a single parameter), these quadratic $J_c(z)$ controlled by the Mandelbrot set M ($c = –2$ being M's left end, its accessory external angle $\xi(–2) = 1/2$, the binary bifurcations 2^n's main series' start at $c_{20} =1/4$). The first implication of such a situation is a significantly enhanced probability of finding the number 2 (& its positive/negative integer powers) over others' in laws of nature, which indeed is seen. Same is evident of approximate formulae correlating physical observables' ratios' numerical values to fractal geometry /Mandelbrot set's features (6). Seen from a distance, J_cs, J_c-convolutions and -composites looked like point particles. The 2D surfaces from iterations $z^{(1, 2)}_{n+1} = R(z^{(1, 2)}_n)$ should likely have had membrane /m-stripe character, further dimensions did evolve, maybe for $z^{(1, 2)}$ fluctuation-amplitude reduction purposes. But "thinking in amplitudes" quite rapidly loses significance in case of (some unknown oscillator's) sophisticated phase modulation, where the entire meaningful information is carried by phase functions and -functionals (the external angles $\xi(c)$ accessory to $c \in \partial M$, Mandelbrot set's boundary, being such $g(f(\varphi))$). We noticed that (linearized around $(x(t), dx(t)/dt)=(0,0)$) Van der Pol oscillator's eigenvalues $(\lambda \pm \sqrt{\lambda^2 – 4})/2$ for $|\lambda| \leq 2$ equal $[–2, 2]$ double line segment's conformal mapping onto the unit circle $|z|=1$.

The second implication of the situation as described is a strong effect on "objects'" ability to interact, due to control of their connectivity by M from its definition $M = \{c \in \mathbb{C}$: Julia set J_c is connected$\}$. As early as in 1924 Weyl (7) stated that one need not further ascribe the connectivity conditions of Euclidian space to space in general relativity theory, i.e. it can be multiply connected (thus contain inner surfaces too, attributed to elementary particles with mass by him). If not space itself but the "objects" hosted /propagating are considered in a connectivity context, a case like this is not unlikely: for c from within an ε-stripe along ∂M – sufficient spectral weight and modulus of rapid fluctuations in c taken for granted – one would see rapid oscillation in connectivity, adjoined Julia set break up into a "cloud of points" (a Cantor set) and gain former connectivity again. For c values out in M's potential region, off the fluctuation range along ∂M, the J_cs are Cantor dusts and thus are unable to propagate fields /oscillations or to couple to such, but – lying in the 2D support surfaces – probably had to (passively) follow planes' local gravitational distortion (the vacuum state for the other interactions) and been able to co/anti-gravitate in a quite exotic manner (via fake /creation of long-lived "open ends" within the membranes' areas, so locally imposing Neumann boundary conditions there). In case of c exceeding 4 in modulus, Mandelbrot set M finally loses its universality, for $c \in M$, |c| is less or equal 2 (2 for M's left end). Thus one could have a look at the area-ratio of permissible c-values producing Cantor dust J_cs or connected ones (for parameter $c \in M$, Mandelbrot set's area A_M being 1.50659177(8)):

$$C(|c|) = 1 - A_M/(|c|^2\pi)\,, \quad (A_M \text{ to be replaced by its share } A_M(|c|) \text{ for } |c|<2) \qquad \text{Eq.(1)}$$

the fraction C(|c|) measuring the Cantor-dust-c area's "percentage", its numerical values thus being C(4) = 0.970027309(2) and C(2) = 0.880109236(6), C(2) not far off estimated dark matter & dark energy's share, the total contribution of matter and energy to ρ_c being within 10% of the critical density (from cosmic microwave background anisotropy data as referred to by Roberts (8) in his comprehensive review of vacuum energy). {1 – C(π)} did reproduce a share of approx. 5% matter and energy and {1 – C(4)}, an expression of *limit character*, such of 3% which might have met the data in our universe's earliest stage. But *initial values* maybe fit de Vaucouleurs' verdict that the numerous attempts at deriving the numerical values of fundamental constants of nature from first principles "have all ended in failure or numerology" (9), as well. But still the iterative access to both, the complex 2D support spaces and "objects' dynamical cores" hosted, these brought into being in a quite erratic manner, should have been to Wheeler's taste, fitting his "events so numerous and so uncoordinated … yet fabricate firm form"… statement (10). According to the iterations' inherent discreteness (their *final results* fulfilling continuum or at least dense "discretuum" demand, latter term borrowed from Bousso & Polchinsky (11)), space-time might acquire a quite discrete character at least as far as most of its space dimensions are concerned. Vacuum's "energy loan", kindly granted to turn virtual "chance events" (12) into real ones, apparently forces universe's closure: The overdraft limit at any $C(|c_{crit.}|) \ll 1$ sometime surpassed, the royal way out of credit crisis is a universe with total energy not even defined.

References /Remarks

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