Mandelbrot set's links to physics: more of same?

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Abstract
Soon after B. Mandelbrot's discovery of the set bearing his name (1), fractal geometry as tool in the phase transition and renormalization context was accepted (e.g. identification of the Yang-Lee zeros in the thermodynamic limit with the Julia set of the renormalization transformation) and since extensively used in physics and synergetics. But, as we see it, too often as mathematically useful procedure than as nature's possible route for evading scale-down toward where all problems are linearized by regression to the infinitesimal(2), via branching off in order to conserve nonlinearity. Lacking a viable *and* rigorous method, we examined fractal geometry's suspected role in scaling electrodynamics' fundamentals by a heuristic bottom-up strategy. Mandelbrot set's rich mathematical structure combined with adjoined (quadratic) Julia sets' dynamics is likely to grant further hypothetical links to fundamental physics (provided that those already traced are not merely accidental). The mathematical foundation of the subjects on the agenda essentially goes back to G. Julia, P.Fatou, B.B. Mandelbrot, J.H. Hubbard, A. Douady, P.J. Myrberg and M.J. Feigenbaum.

Keywords: Fractal geometry, Mandelbrot set, external angles, Julia sets, Cantor set, Myrberg-Feigenbaum point, period doubling oscillations, control space, connectedness, Planck mass scale, gravitation

Introduction
Deviating from fractal geometry's use as a mere mathematical tool, making direct contact of Mandelbrot set M = {$c \in \mathbb{C}$: Julia set J_c is connected} 's features, e.g. external angles (3) $\xi(c)$ in Hubbard's algorithm, and bifurcation data etc. to physical observables was tried in a tentative and heuristic manner (4, 5). So, approximate formulae were found for $\alpha(0)$, the infinite distance limit of electrodynamics' effective coupling constant $\alpha(\kappa)$, and for M_P/m_e, the Planck mass to electron rest mass ratio (getting fit precision enhanced by an order of magnitude when using α's CODATA 2002 value instead of the approximated $\alpha(0)$), or its generalization to M_P/m_i, i = e, μ and τ, the 1st, 2nd and 3rd generation charged leptons.

In a more qualitative manner, access to fractional specific charge, likely tied to external angles $\xi(c_k)$ accessory to the main series of period doublings on M's real c-axis, and to specific rest mass (better a function of $1/m_{sp}$, this seemingly tied to external angles $\xi(c)$, $c \in [-2, c_D]$, to the Myrberg-Feigenbaum point's left), was probably found. Taken together, this seems to mirror phases' (or ΔΦ's) *exceptional* significance (6) in structuring matter.

Links to the things?
Generally, expecting observable effects from Mandelbrot set's action as control space (3) (if any), is not out of place. Belonging to the iterative $z \rightarrow z^2+c$ map, M would not lose its significance (at least locally) for non-polynomial maps because of its universality, and it likely had enough unifying power, because everything depends on a single parameter, c.

If period doubling oscillations played a role in structuring the generalized charge space of internal symmetries and nature behaved according to the main series of period doublings on the real c-axis of the Mandelbrot set, starting from period 2^0 - oscillations accessory to integer charged states, $N=2^k+1$ constituents $((2^k+1)$ is the denominator of external angles $\xi(c_{2k})$ with $k = 2^n$, $n = 0,1,2,\dots)$ most likely formed one particle of period k - oscillation, this being "neutral" with respect to the charges associated with the 2k-force.Thus, quark(k=2) substructure would then comprise 5 period 4 - particles (likely 't Hooft's "quinks"(7)), each of these 17 period 8 - particles (the "teens", each of which then contained 257 period 16 - particles)… The question is how far this probably goes down. Due to the 1 - 2D situation, one starts from a relatively high level of internal parameters' fluctuations, especially near bifurcation roots c_k. In addition, bifurcations and fluctuations use to interact, at least on a macroscopic scale. Final termination of the series of binary bifurcations might come from (external?)metric fluctuations (or better from their retroaction upon the rapidly converging bifurcation root distances $|c_{2k} – c_k|$, which can be interpreted as phase functions, and on external angles, then being phase functionals). The fact that not an arbitrary, large mass^2 cutoff or such around GUTs' scale mass squared (8) showed up in the α(0) approximation (4), but a value of order Planck mass squared (in a ~ $\log^2(C.M_P{}^2/m_e{}^2)$ term, rewritten from mass ratio) might signal gravity's role in closing the game. A scale breakdown obliterated form, such strip-off seemingly being necessary for a constituent to be truly elementary, at least in a geometrical sense (for smallest spatial entities'("hodon") properties see (9, 10)). Furthermore, gravitational interactions had to be included anyway, because constraining of the weak-electromagnetic couplings would require a theory that unites all forces (11).

So far, only a fraction of (main) Mandelbrot set's features (main series of bifurcations on the real c-axis, the accessory external angles and such belonging to real c-values $\in [–2, c_D]$) was considered. From the hypothetical period doubling oscillation ↔ particle duality as described before, the same pattern for 2nd generation particles and such oscillations, this time belonging to the (main) bifurcation series (on the real c-axis) of the *secondary* Mandelbrot set, seemed to suggest itself. But such analogy did not work, and fractional specific charge^2 came out wrong, the whole situation being intricate in all probability (5).

Control on connectedness of Julia sets J_c accessory to the iterative $z\rightarrow z^2+c$ map, right from the Mandelbrot set's definition, might be another efficient tool if nature indeed made use of M as control space (thus living separation of powers, execution of law transferred to Julia sets which carry the dynamics). For parameter c leaving ∂M, the Mandelbrot set's boundary, towards the potential region outside, the accompanying connected Julia set J_c decayed into a Cantor set, i.e. a cloud of disconnected points. Given a spectral weight to rapid fluctuations in c, at bifurcation roots c_k on M's real c-axis an infinitesimal imaginary part ε (at c_1 a real-part-increase ε) would lead to oscillatory "connected → disconnected" and reverse digital transitions. If sufficiently fast in $\langle\omega^2\rangle$, they maybe guaranteed charge cohesion, not automatically granted unlike for mass. At the fundamental level, one likely is unable to discriminate whether the (J_c-related)"object" lost connectivity or the scale did.

Conclusions

In line with earlier work, further possible relations between fractal geometry and physics have been considered heuristically. These hypothetical links were remarkable and likely made physics inexhaustible, if nature really used the Mandelbrot set as a control space.

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