Short note on the particle generations problem

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Abstract
Within the possible role of fractal geometry in scaling electrodynamics' fundamentals, the "superfluous leptons" part of the particle generations problem is touched upon, as far as the mass scale is concerned.

Keywords: Particle generations, mass scale, muon, tau, period doubling, Mandelbrot set, external angles

Introduction
One still does not know the reason for the existence of extra elementary particle families. The old electron-muon puzzle evolved into a puzzle of apparently redundant generations of both leptons and quarks (see e.g. (1)). Regardless whether the extra families are just heavier than the first one, "phenomenologically irrelevant supercargo" as was argued by Glashow (2), or somehow significant after all, the regarded muon and tau particles would likely be affected as is the electron in case of suspected fine-tuning of the mass scale by fractal geometry.

Mass scale
A heuristic fit procedure (3), using the CODATA 2002 values for the Planck mass M_p and the electron mass m_e, yielded the following approximation (to 6ppm) for the masses ratio:

$$\frac{M_P}{2m_e} \approx \frac{\sqrt{2}\,\ln(\delta_{2D})}{\sqrt{\pi P}|c_D|\ln(\delta)}\exp(\gamma^{1/2}e^{\pi/2+1/2}\pi^{e/2+1/2}), \qquad \text{Eq.(1)}$$

where γ is the Euler-Mascheroni constant, P the Thue-Morse constant, c_D the Myrberg-Feigenbaum point's coordinate, δ Feigenbaum's universal number and δ_{2D} Feigenbaum's constant for an area-preserving 2-dimensional map (4, 5). The result might be accidental, but if there is something in it, one probably could expect a similar expression for charged 2nd and 3rd generation leptons. Generalization of Eq.(1) by using – $d/dz(\Gamma(z))$ around z=1 instead of γ and additional prefactors (like $\gamma^{1/2}/|\Gamma'(z)|^{1/2}$ or its square etc.) did not work, but change of the e↔π dual expression to $e^{B(1/2,\, f(z))/2+1/2}(B(½, f(z)))^{e/2+1/2}$, B being Euler's Beta function, finally did. Following the $\sqrt{\pi P}/2$ reformulation in terms of the Mandelbrot set's (6) real c-axis values and accessory external angles (7) ξ(c) from (3), $\Gamma(\xi(-2))\sqrt{\xi(-2)\xi(c_D)}$, the more general form of the M_P/m_i approximation, i = e, μ and τ, suggests itself as

$$\frac{M_P}{m_i} \approx \frac{2\ln(\delta_{2D})}{\sqrt{\xi_{oi}P}\,\Gamma(\xi_{oi})|c_D|\ln(\delta)}\exp(\gamma^{1/2}e^{B(1/2,\, \xi_i)/2+1/2}(B(½, \xi_i))^{e/2+1/2}), \qquad \text{Eq.(2)}$$

$$\text{with } \xi_i = \left(1/2 + \frac{\pi^2\, \Gamma^2(\pi/2+1/2)\, \xi_{oi}^2}{4\Gamma((e/2+1/2)\ln(\pi))\, \Gamma(1/2 - \xi_{oi})}\right). \qquad \text{Eq.(3)}$$

For $\xi_{oi} = 1/2$, i.e. in case of the electron, the approximation Eq.(2) reduces to Eq.(1). One tentatively could perform a rotation in external angle by $-(1/2 - P)$, so $\xi_o(c)$, seemingly tied to specific mass, comes to coincide with the infinite k-limit of $\xi_o(c_k)$, accessory to the main series of period doublings on the real c-axis of the Mandelbrot set M and somehow tied to specific charge. So, $\xi_o(c_D)$ probably is to be identified as $\xi_{o\tau}$. Partial rotation could lead to $\xi_{o\mu}$, coming as the geometric mean of 1/2 and P. Using $\xi_{o\mu} = \sqrt{P/2}$, $\xi_{o\tau} = P$ and the CODATA 2002 values for the Planck, muon and tau particle masses, Eq.(3) plus Eq.(2) yield M_P/m_i approximations to 40ppm for the tau particle and to 635ppm for the muon.

Small changes in specific angle combined with $\xi(\xi_o)$ and $B(½, \xi)$ functional dependences could be responsible for the relatively small charged lepton mass differences (compared to the huge mass scale corresponding to their pointlikeness), provided this interpretation is essentially correct. The repetitive particle family pattern might go back to approximately self-similar structures, maybe involving (quadratic) Julia sets, which would come naturally and carry dynamics under the Mandelbrot set's control when acting as control space.

Conclusions

Generalized, our heuristic results concerning a possible role of fractal geometry in charge quantization and electron rest mass fine-tuning seem to be applicable to the 2nd and 3rd generation charged leptons. Unfortunately, no significant contribution towards solving the particle generations puzzle can be offered.

References

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