Thue- Morse constant approximants' use in fitting physical observables ratios' numerical values

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Abstract
As an update of our previous heuristic and tentative work on the possible role of fractal geometry (in a more general sense) in scaling electrodynamics' fundamentals (see the physics files of our contentious results website, http://culetto.at/private_research_associates/ …), recent progress in improvement of the former approximation formulas' quality of fit (AFQF) is reported.

AFQF – enhancement, regardless of whether true or accidental relation
As a crude approximation (i.e. to 63ppm) to the *electron rest mass – dressed "indivisible" entity rest mass* ratio's numerical value $m_e/\langle m_0\rangle = 2.339\,112\,29\ldots\mathrm{x}10^6$ (the entity spoken of with almost no features of its own, dressed by the electroweak, strong and further four even stronger interactions/forces, gravity included), Eq.(1) was found

$$\frac{m_e}{\langle m_o\rangle} \approx \frac{2P\sqrt{P}}{2^{2P}}\,\frac{\ln(\delta)^2}{\ln(2)}\,(\exp(P^{1/2}e^{\pi/2}\pi^{e/2}))\ , \qquad \text{Eq.(1)}$$

(http://culetto.at/private_research_associates/sciencephilosophy7.pdf), where P is the Thue-Morse constant and δ Feigenbaum's universal number. When stopping the period doubling (c_ks of the main sequence on Mandelbrot set's real c-axis) at the 4th bifurcation (with accessory upper external angle $\xi(c_{24})=106/257$ as n=4 approximant to P) instead of going to the infinite-k limit of the (upper) external angles ending up with P, Eq.(1)'s AFQF can be improved to 9.4ppm, the fit value got been $2.339\,134\ldots\mathrm{x}10^6$. By a trial-and-error method testing of the exp( )-function's pre-factor, the optimum fit formula got thus reads

$$\frac{m_e}{\langle m_o\rangle} \approx \frac{(4+2c_D)}{(4+c_D)}\,\frac{\ln(\delta)}{\ln(2)}\,(\exp(P_4^{1/2}e^{\pi/2}\pi^{e/2}))\ ,\ P_4=106/257, \qquad \text{Eq.(2)}$$

where c_D is the Myrberg-Feigenbaum point's coordinate, and Eq.(2)'s AFQF is 0.23ppm, the fit value got been $2.339\,112\,84\ldots\mathrm{x}10^6$. The (formal) masses ratio's shape apparently gives a "log-potentials"-ratio more understandable (formally in line with the $\ln(\delta_{2D})/\ln(\delta)$ one of the Planck mass – electron mass ratio approximation given in sciencephilosophy. pdf), Eq.(2) also containing |c|= 4, the maximum in modulus of c up to which Mandelbrot set M's universality is guaranteed.

And for the *proton – electron rest mass* ratio (see the sciencephilosophy7 file, Eq.2),

$$\frac{m_p}{m_e} \approx \frac{P_4\ln(\delta_{2D})}{c_D\ln(\delta)}\left(\frac{1}{\gamma\ln(2\pi\delta^2\alpha(0;P_4))} - \frac{\pi P_4}{2}\right)\ , \qquad \text{Eq.(3)}$$

P_4 being the n=4 approximant to P, δ_{2D} Feigenbaum's number for an area-preserving 2D-mapping (Tabor, M. Chaos and Integrability in Nonlinear Dynamics: An Introduction, 225 Wiley, New York, 1989; Weisstein, Eric W. "Feigenbaum Constant". From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/FeigenbaumConstant.html), γ the Euler-Mascheroni constant and $\alpha(0;P_4)$ the fine-structure constant α's (sciencephilosophy file's Eq.1, P replaced by P_4) approximated value, the AFQF gets 0.12ppm with respect to the m_p/m_e CODATA 2010 value. Same vice versa is true of Eq.(3)'s version nearer to number theory (see sciencephilosophy5 and 7 files' Eq.1, now replacing P there by P_4),

$$\frac{m_p}{m_e} \approx \frac{P_4\ln(\delta_{2D})}{|c_D|\ln(\delta)}\,(e^{\pi+1}\pi^{e+1}) \;, \qquad \text{Eq.(4)}$$

the fit value got been 1836.152 $\underline{4}$54…compared to 1836.152 672…(from CODATA 2010). In case of $\alpha(0;P)$'s (=$\alpha(0)$ of the sciencephilosophy file, Eq.1) use instead of the $\alpha(0;P_4)$ approximant, Eq.(3)'s fit value stays unchanged within the AFQF granted, the ratio got been 1836.152 $\underline{4}$52…And the *fine-structure constant* $\alpha(0)$'s (sciencephilosophy.pdf, Eq.1) approximated value from

$$\alpha(0) \approx \frac{1}{2\pi\delta^2}\left(\exp\left(-\frac{1}{\gamma(e^{\pi+1}\pi^{e+1}-\pi P_4/2)}\right)\right) \;, \qquad \text{Eq.(5)}$$

P_4 again being the n=4 approximant to P, is 7.297 352 568 7…x10^{-3} compared with its 7.297 352 5698(24) x10^{-3} CODATA/NIST 2010 value. Unfortunately, any indications that bifurcations with $n > 4$ indeed could be inactive/ignored in $\alpha(0)$ fine-tuning are still lacking. Furthermore, the *semi-Planck mass – electron rest mass ratio* approximation (its original version in sciencephilosophy.pdf, Eq.2) after replacement of the Thue-Morse constant by its n=4 approximant reads

$$\frac{M_P}{2m_e} \approx \frac{\sqrt{2}\,\ln(\delta_{2D})}{\sqrt{\pi P_4}\,|c_D|\ln(\delta)}\,(\exp(\gamma^{1/2}e^{\pi/2+1/2}\pi^{e/2+1/2})) \;, \qquad \text{Eq.(6)}$$

which is 1.194 6$\underline{3}$1…x10^{22} compared with the masses ratio's value of 1.194 652…x10^{22} from their CODATA 2010 values. Again, $n > 4$ bifurcations' role (if any) is open. Eq.(6) in terms of $\alpha(0;P_4)$ (sciencephilosophy Eq.3) with CODATA 2010 α gives 1.194 6$\underline{4}$2…x10^{22}.

And finally the *Planck mass – proton rest mass ratio* approximation (sciencephilosophy5, Eq.2), when using the n=4 approximant to P reads

$$\frac{M_P}{m_p} \approx \frac{1}{\sqrt{\pi P_4^3/8}}\,(\exp(\gamma^{1/2}e^{\pi/2+1/2}\pi^{e/2+1/2}-(\pi+1+(e+1)\ln(\pi)))) \;, \qquad \text{Eq.(7)}$$

which gives 1.301 2$\underline{3}$2…x10^{19} compared with 1.301 256…x10^{19} , the numerical value of the ratio calculated from the CODATA 2010 M_P and m_p values.