Does fractal geometry tune electrodynamics' scales?

F.J. Culetto and W. Culetto*)
Private Research Associates, Stallhofen 59-60, A-9821 Obervellach, Austria
*Electronic address: werner.culetto@inode.at
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Abstract
The possible role of fractal geometry in scaling electrodynamics' fundamentals is traced in a tentative, heuristic and also constructivistic manner. Mandelbrot set (1) features, e.g. external arguments and bifurcation parameters are considered candidate to maybe end up in laws of nature.

Keywords: Coupling constant, period doubling, Mandelbrot set, external arguments/angles, mass scale

Preface
In 1909 Albert Einstein found that the dimension of e^2/c (= elementary charge squared divided by speed of light) is an action as is true for Planck's constant h. So, he thought that there must be a connection. Changed to $\alpha=2\pi e^2/hc$ (in cgs-units), the dimensionless ratio (≈ 1/137) named fine-structure constant was introduced by A. Sommerfeld in 1916 when solving Kepler's problem in Bohr's theory, including relativistic dependence of mass on velocity. Dirac theory's (1928) formula for energy eigenvalues of the hydrogen atom still contains α in its original context. Quantum electrodynamics (QED) finally allowed for calculation of radiative corrections (1949), so turning α's nature to that of electromagnetic force's accidentally (?) small coupling constant, which weakly depends on the energy κ where it is measured. Its value $\alpha(0)=0.007297352568(24)$ (2002 CODATA), coming from comparison of experimental and QED results for electron's anomalous magnetic moment (α(0) was confirmed independently by quantum Hall effect precision measurements too), is amongst the most precisely known quantities encountered in physics.

Still adhering to the Neo-Einsteinian conviction, one could ask for the fate of the $e^2/c \leftrightarrow \hbar$ connection (if any), and also in this context, if indeed all consequences of the quadratic relativistic Hamiltonian (and the S-matrix, yielding probabilities by squaring amplitudes) did already manifest themselves. Or looked at the other way round, living with nonlinear complex dynamics, does the Mandelbrot set (which belongs to the iterative $z \rightarrow z^2+c$ map) yield observable effect(s), if acting as control space (2)? Our quite speculative paper (its results could be accidental after all) tries to find such evidence or construct a way things could be organized, respectively. If the α(0) approximation Eq.(1) more or less reflected reality, whatever this is, electromagnetic force's coupling couldn't escape geometrization. Because physical objects are seldom self-similar over more than 4 orders of magnitude, the $z_{n+1} = z_n^2+c$ process (the Mandelbrot set M is obtained by fixing $z_0 = 0$ and varying the c parameter) is not considered a proper description of the real world (3). But here one gets one's sums wrong, disregarding Mandelbrot set M's structural stability. One inherits all its combinatorial features, when iterating (holomorphic) functions, which just (locally) had to roughly resemble $z \rightarrow z^2+c$ after proper rescaling (2)).

Coupling constant

Involvement in the theme was quite accidental. By playing with numbers (and educated guessing, later in 2005) we found a strange approximation (Eq.(1)) to the fine-structure constant $\alpha(0)$ with ingredients from fractal geometry. So far this was nothing special, just one more try in α-numerology, e.g. see (4). But what did really amaze us, was the likely embeddings of relating the infinite distance limit of electrodynamics' "running" coupling constant to some underlying principle. The expression, resembling Gaussian distribution density squared $\varphi^2(\sigma X, \sigma=\delta)$, almost at maximum amplitude, reads

$$\alpha(0) \approx \frac{1}{2\pi\delta^2}\left[\exp\left(-\frac{1}{\gamma(e^{\pi+1}\pi^{e+1}-\pi P/2)}\right)\right], \qquad \text{Eq.(1)}$$

(or with a + P/2 term instead of $-\pi P/2$ in its expanded, linearized and modified version), where δ is Feigenbaum's universal number, γ the Euler-Mascheroni constant and P the Thue-Morse constant. This might be pure coincidence, but if there is something in it, the result, giving $|\alpha_{approx.} - \alpha_{exp.}(2002\ \text{CODATA})| \leq 7.\ 10^{-12}$ (latter value for the less accurate, linear version), likely comes from a 1 - 2D nonlinear dynamical problem including period doubling oscillations, regarded in the infinite distance and bifurcation limits (and maybe infinitesimal limit(s) too). Whole truth could come out far more sophisticated, the problem not being restricted to the rest and infinite momentum frames, but demanding correct $\alpha(\kappa)$ for physics in between. So one might have to handle a scenario of complex (likely fractal, hopefully not wildly mixing) fluxes' ratios for continuous κ, regarded in the various limits, the infinite distance limit $\alpha(0)$ finally turning out 1/4 of the electric to magnetic integer flux quanta ratio. Function $\alpha(\kappa)$ had to ± satisfy the accessory renormalization group equation. All reasoning is restricted to recent time intervals, very small compared to the age of the universe, so excluding all aspects of α-variability (see e.g. (6)) other than κ dependence.

Thue-Morse sequence's involvement pointed towards a digital regime at the infinitesimal level. Problem's obvious complexity demanded a synergetics (5) type approach (possibly benefiting from likely good manners of "aggregated" variables), but as matters stand, we have to cope with a heuristic, trial-and-error fit procedure at the moment.

(Fractional) charge

Once seen colour pictures of Mandelbrot set potential details, one cannot longer believe that objects of this beauty and complexity stayed without a bond to reality. Here, Adrien Douady's dictum (that the sophisticated combinatorial features of the Mandelbrot set M seem to indicate) "that external arguments are not just a mathematician's trick, a useful artefact, but that they really occur "in nature" "(2), could maybe get some support. In 2D-electrostatic analogy, you see, field line starting from point x_0 of ∂M, the boundary of the Mandelbrot set, finally reaches a point x_∞ on a ring electrode at infinity, latter point here characterized by an angle, called external argument ($2\pi\xi, 0\leq\xi<1$) of x_0 with respect to M. Electrostatic analogy, which is known to work well for reproduction of interacting strings' topology by topology of equipotential lines (7), could be usefully extended by thinking of field lines' bond to "aggregated", deeper level variables. Indeed, the so-called external angles $\xi(c_2) = 1/3$ and $2/3$, accessory to the first bifurcation of the main series of period doublings on the real c-axis of the Mandelbrot set M, coincide with the absolute values of quark (electric)charge quantum numbers. For $c \rightarrow c_D$, the Myrberg-Feigenbaum point, i.e. in the limit of infinite bifurcation, the (upper) external angles are known to converge to the Thue-Morse constant P. Whatever exact relation between external angle $\xi(c)$ and particle fractional charge (or partially aggregated variables of underlying collective phenomena to blame for fractional flux quantization, as could be multi-stage magnetic superconductivity) might hold for higher bifurcations and in the infinite bifurcation limit, the $-\gamma\pi P/2$ (or $\gamma P/2$) small corrections in Eq.(1), if not just accidental, likely emerged from (ratio of) quantities tied to charges. Finally, for period 2^0 - oscillations, the absolute values of leptons charge quantum numbers coincide with $\xi(c_1) = 0$ of $c_1 = 1/4$, cusp of the big Mandelbrot cardioïd, specific angles being counted modulo 1. If these coincidences are not accidental, kind of control on asymptotic properties (of variables $|Y_i|$, $\bar{Y}_i Y_i$ & Y_i^2) seems to be exerted by M.

If nature simply followed the considered bifurcation path right to the limit, inflation of new particles and forces seems inevitable. Particles, at present considered elementary, may turn out composite again, as was argued by 't Hooft and Ne'eman (8) long time ago. So, $N = (2^k+1)$ constituents (2^k+1 is the denominator of external angles $\xi(c_{2k})$ with $k = 2^n$, $n = 0,1,2,\ldots n_{crit.}(?)$) most likely formed one particle of period k - oscillation, this being "neutral" with respect to the charges associated with the 2k-force. Quark (k=2) substructure would then comprise 5 period 4 - particles, each of these 17 period 8 - particles…One can just speculate how far this could go down. However, for desirable unification of forces (if still possible), the "grand" gauge group G would then end up quite far off the minimal group SU(3) x SU(2) x U(1). Huge complexity of ∂M near bifurcation roots c_k, just slightly off the real axis, would maybe allow for "escape" above some critical period 2^n - oscillation, or grant "damping" effects and account for a series of transient phenomena, respectively. Quantum phase, retroacting on the real c-axis segment via $z \rightarrow \sim(z+1/z)$ conformal map, likely required inclusion of infinitesimal limit(s) too (like $(c-c_k) \rightarrow 0$ at each k for whatever function or ratio $F(z',z'_k,\xi(z'_k)\ldots)$, or even $(c-c_0) \rightarrow 0$ limits for every c_0 from $[c_D,1/4]$) when deriving exact expression for $\alpha(\kappa)$.

Mass scale

Aside maybe charge-internal corrections, Eq.(1) contains a term likely tied to polarization, [1 – its inverse], an approximate charge^2 correction factor, being quite unlike QED's one. Expecting log-expressions with large mass^2 cut-off, sum total(?) would differ from these. Vacuum polarization at ultra-high energy level needn't fit extrapolated low energy results, but one made an even worse mistake by tying such complex matter to an approximation. Let Z_c^2 be the $e\leftrightarrow\pi$ dual large correction term of Eq.(1), $Z_c^2 = \gamma e^{\pi+1}\pi^{e+1}$, then one should worry about its very origin or that of Z_c, respectively. Amazingly, $\exp(\gamma^{1/2}e^{\pi/2+1/2}\pi^{e/2+1/2})$, an approximate(?)limit expression, is within the order of Planck mass M_P divided by twice the electron mass. Instead of $\gamma^{1/2}$, usage of $z^{1/2}$ from $e^{z+1}z^{e+1} = \pi P/2$ gave approximation to around − 3%. But the fit could easily be made almost perfect, e.g. by finding a logically plausible pre-factor A to $\exp(|Z_c|)$, best being a ratio of limit quantities. If one took P, the external angle tied to the infinite bifurcation limit, formed pre-factor $_{(\pm)}P/(2^P-1)$ and used the 2002 CODATA values for M_P and m_e, one would get a deviation of the approximated $M_P/2m_e$ ratio (= $P\exp(|Z_c|)/(2^P-1)$ in this case) from the experimental one of about 0.01%. In the k = 1 limit of $\xi(c_k)$, $\xi(c)/(2^{\xi(c)}-1)$, giving $P/(2^P-1)$ for infinite k, converged to $1/\ln(2)$.

Several hundred combinations of period-doubling specific variables or ratios were tested. Best fit (to 6ppm) was obtained choosing the pre-factor $(2/(\pi P))^{1/2}\ln(\delta_{2D})/(|c_D|\ln(\delta))$, a less accurate value (to 130ppm) with $\Gamma^{1/2}(\gamma)\ln(\delta_{2D})/(|c_D|\ln(\delta))$. Both A-expressions contain δ_{2D}, Feigenbaum's number for an area-preserving 2-dimensional map (9, 10), so reflecting 2D character of mass. Reinserting the approximated $\alpha(0)$ into the mass ratio approximation with any good fit pre-factor A then would give $M_P/m_e \approx 2A\exp((\gamma\pi P/2-1/\ln(2\pi\delta^2\alpha(0)))^{1/2})$. At maximum simplicity, pretty good fit for mass fine-tuning could be obtained by taking $\sqrt{A^2(\xi(c))}$ in the Myrberg-Feigenbaum limit c_D. But much better fit, welcome degrees of freedom for meeting correct $\alpha(\kappa)$ and the whole problem's degree of sophistication clearly favour the use of $A(c, c_k, \xi(c), \xi(c_k), \delta_k, \delta_{2D,k})$ in the infinite k limit, thus yielding the ratio

$$\frac{M_P}{2m_e} \approx \frac{\sqrt{2}\ln(\delta_{2D})}{\sqrt{\pi P}|c_D|\ln(\delta)} \exp(\gamma^{1/2}e^{\pi/2+1/2}\pi^{e/2+1/2}). \qquad \text{Eq.(2)}$$

Finally, "log-potentials" matched 2D situation, and exp(exp( )) allows for the huge scale. Rewritten in terms of external angles and Mandelbrot set's real c-axis values, $\sqrt{\pi P/2}$ were $\Gamma(\xi(-2))\sqrt{\xi(-2)\xi(c_D)}$, containing the geometric mean of the external angles accessory to $c=-2$, left end of M, or c_D, the Myrberg-Feigenbaum point, respectively. Approximation Eq.(2) can be rewritten too, using $\alpha(0)$ from its approximation (Eq.1), yielding

$$\frac{M_P}{2m_e} \approx \frac{\sqrt{2}\ln(\delta_{2D})}{\sqrt{\pi P}|c_D|\ln(\delta)} \exp((\gamma\pi P/2-1/\ln(2\pi\delta^2\alpha(0)))^{1/2}). \qquad \text{Eq.(3)}$$

Much less accurate than for $\alpha(0)$, but nevertheless with adequate precision, electron rest masses approximate position on the Planck mass scale could in principle be reproduced without using "unnatural" variables, not yet available from the nonlinear dynamics' store. But one has to accept deductionists' aversion to both constructivistic (or even considered autistic) bottom-up approaches. Thus, we regret having only been able to offer a simple, tentative procedure plagued by a high degree of arbitrariness. Future etiology of why/how electrodynamics' fundamentals acquire the values they happen to have, might well some day confirm the period-doubling specific variables' impact.

Conclusions

Concluding, there seem to be indications that electron rest mass fine-tuning and charge quantization might be tied to fractal geometry. At least, such bond could be constructed. There indeed could be good reasons for concentration upon nonlinear dynamics (in 1D and 2D) of suitable objects (if appearing in the six and more extra-dimensions to space-time context or elsewhere), the tools nature might have to cope with for agglomeration of what we "macroscopically" use to call charge and mass. Much arbitrariness still prevents one from significant statements, for instance that the Mandelbrot set M, lacking dynamics of its own, *really acts* on the physical world as kind of control space.

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