$\alpha(0)$ (fine-structure constant) - approximation formula: Update

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Within our 2006 tentative, heuristic and also constructivistic procedure*) in tracing a possible role of fractal geometry (this understood in a broader sense) in setting the values which electrodynamics' fundamentals happen to have, the following (prototype) approximation to the *fine-structure constant α(0)* was found

$$\alpha(0) \approx \frac{1}{2\pi\delta^2}\left[\exp\left(-\frac{1}{\gamma(e^{\pi+1}\pi^{e+1} - \pi P/2)}\right)\right], \qquad \text{Eq.(1)}$$

where $\alpha(0)$ is the infinite distance limit of electrodynamics' running coupling constant, δ Feigenbaum's universal number, γ the Euler-Mascheroni constant and P the Thue-Morse constant. There had been some concern about the exp-function argument's denominator of $(Z_c^2 - Z_{c\ small}^2)$ shape, as to the completeness of $Z_{c\ small}^2 = \gamma\pi P/2$, small correction term there. As it contains the $\xi^{inf}(-2)\xi^{inf}(c_D)$ product of the external angles accessory to the Mandelbrot set's left end −2 and to its main bifurcation series' Myrberg-Feigenbaum point c_D (= – 1.401155... + 0i) , a maybe phenomenological formulation of some relative mass ↔ relative charge interplay, the possibility of a corresponding near-field-term product's occurrence, i.e. of $\xi^0(-2)\xi^0(c_D)$ was then mentioned in the erratum_sciencephilosophycomments.pdf **) file of our www.culetto.at website. As an alternative − this considering the iterative nature of the basic complex quadratic map, with just both dimensionless variable z and parameter c permissible, and so zero point energies quantum number counted − Zeta-function regularization (1, 2) of the infinite valued $\frac{1}{2}\sum_n n_{zero}\sum_n n_{period}$ product would contribute (–1/12)(–1/12), thus $Z^{eff}{}_{c\ small}^2 = \gamma\pi P/2 + 1/288$). Eventually, the small correction's additive term, then expected to be << $\gamma\pi P/2$, indeed is.

Inserting this effective expression instead of $Z_{c\ small}^2$ into approximation Eq.(1) yields $\alpha(0)_{approx}$ = 0.0072973525652 , versus the measured α = 0.0072973525649(8) (α^{-1} = 137.035999166(02)(15)) published in 2023 by Fan et al. (2). And the available NIST/CODATA 2022 α-value is 0.0072973525643(11).

1) Kaku, M. *Introduction to Superstrings and M-Theory*, Second Edition, 439 (Springer-Verlag, New York, 1999)

2) Moreta, J.J.G. The Application of Zeta-Regularization Method to the Calculation of Certain Divergent Series and Integrals. Prespacetime Journal 4(8), pp 213-226 (2013)

3) Fan, X., Myers, T.G., Sukra, B.A.D., Gabrielse, G. Measurement of the Electron Magnetic Moment. Phys. Rev. Lett. **130**, 071801 (2023); arXiv: 2209.13084v2

*) Culetto, F.J., Culetto, W. Does fractal geometry tune electrodynamics' scales? www.culetto.at/private_research_associates/sciencephilosophy.pdf (2006)

**) The conversion error symbol □ there meaning a minus sign