Appendix (F1): Internal supplement (Aug. 2020): Hausdorff dimension and particle motion

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Within our tentative description of all things / structures and the dynamics there are by using Julia set fractals J and their control spaces / connectedness loci (for the second order polynomials iterated, the main and midget Mandelbrot M sets), J(z)'s effective Hausdorff dimension $D_H^{eff}(J)$, $1 \le D_H^{eff}(J) \le 2$, carries relative information on particles' motion and mass. As the Js are subsets of the plane, a simple method for restricting $D_H(J(z))$ to [1, 2] in all cases of non-pathological relative motion would be its tentative, exchange-symmetric (abbreviated) formulation as

$$D_H^{res} = \frac{D_H^{(p)} D_H^{(sys)}}{1 + (D_H^{(p)} - 1)(D_H^{(sys)} - 1)} \quad , \qquad \text{Eq.(1)}$$

where (p) denotes the moving (quantum) particle. For $(D_H^{(p;\, sys)} - 1) = \beta^{(p;\, sys)}$, the composition law of velocities in SR is recoverable, as was shown in the Appendix (E) presented on our www.culetto.at website. In this case, the masses ratio m/m_0 and its Taylor-series expansion would be

$$\frac{m}{m_0} = \frac{1}{[D_H^{(p)}(2 - D_H^{(p)})]^{1/2}} = 1 + (D_H^{(p)} - 1)^2/2 + \text{higher-order-in-}\beta \text{ terms} \qquad \text{Eq.(2)}$$

In case of small complex λ, the Hausdorff dimension of the Julia set J belonging to the iterative $z \rightarrow z^q + \lambda$ map was calculated by D. Ruelle (1982, Ergod.Th. & Dyn. Syst. 2, 99 -107), and – converted to our quadratic-$J_{\mathbb{C}}$ situation ($t = D_H(J_{\mathbb{C}})$, $q = 2$ and $\lambda = \mathbb{C}$) – is

$$D_H(J_{\mathbb{C}}) = 1 + \frac{|\mathbb{C}|^2}{4\ln(2)} + \text{higher-order-in-}\mathbb{C} \text{ terms} \qquad \text{Eq.(3)}$$

As initially thought (and expressed in our Appendix (B)), for the *low velocity regime* of classical dynamics, $m/m_0 \approx D_H(J_{\mathbb{C}})$ could have been the likely simplest relation/ansatz. But as given in Eq.(2), $m/m_0 = f(D_H(J_{\mathbb{C}}))$ is not that simple, SR's β would thus result in

$$D_H(J_{\mathbb{C}}) - 1 = \frac{|\mathbb{C}|^2}{4\ln(2)} + \text{higher-order-in-}\mathbb{C} \text{ terms} \,, \qquad \text{Eq.(4)}$$

the term quadratic in $\mathbb{C}$ now being the right formulation of our *Dorfer's β* suggestion.