Appendix(C) concerning special Julia sets' Hausdorff dimension

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Abstract
The appendix C is a resumption of results/statements of the Appendix B (see the website http://culetto.at/private_research_associates/appendixB.pdf) and contains (our) recent considerations on a possible role of (quadratic) Julia sets' Hausdorff dimension within the (special) relativistic domain of particle dynamics.

Contents
As was already stated in the cited file, there are indications that there might be a relation between the Hausdorff dimension of Julia sets $J_{\mathbb{C}}(z)$ (belonging to the iterative $z \rightarrow z^2 + \mathbb{C}$ map) and quantum particle dynamics' relative energy E/E_o . In the relatively low particle velocity limit, $E/E_o \approx D_H(J_{\mathbb{C}})$ in case of $\beta = |\mathbb{C}|/\sqrt{2\ln(2)}$, $D_H(J_0)$ being 1. Eventually, for the relativistic domain, relative energy E/E_o likely becomes a function of $J_{\mathbb{C}}$'s Hausdorff-dim (fulfilling the continuity demand), the maximum in dim being 2 as all fractals involved are subsets of the plane. We are not aware of any statement A. Einstein might have made concerning the pole in the Lorentz-Einstein equation for the relativistic mass dependence on particle velocity v at $\beta = 1$, who usually used to be very concerned about singularities, especially with field equations. From our tentative fractal dimensional considerations, the appearance of such a pole is just one of at least two possible cases, the other being kind of a "touch down" at ultra-high particle momentum (likely somewhere around $M_P c$, where M_p is the Planck mass), at Hausdorff-dim 2 or little less at $D_H(J_{\mathbb{C}}) = 2 - \varepsilon$, $\varepsilon > 0$.

In search of a function whose low momentum expansion yields $D_H(\mathbb{C})$, the simple ansatz $f(D_H(\mathbb{C})) = 1/(2 - D_H(\mathbb{C}))$ $(= 1/(1 - (D_H(\mathbb{C}) - 1))$ was chosen, $1 \leq D_H(\mathbb{C}) \leq 2 - \varepsilon$, giving E/E_o for sufficiently small $|\mathbb{C}|$. Aside the pole at 2, D. Ruelle's result for $D_H(J_{\mathbb{C}})$ (1982, Ergod.Th. & Dyn. Syst. 2, 99 -107) in case of small complex $\mathbb{C}$, recast as $D_H(J_{\mathbb{C}}) \approx 1+ |\mathbb{C}|^2/(4\ln(2))$, was used for the approximation of its derivative $dD_H(\mathbb{C})/d\mathbb{C}$ – abbreviated $D_H'(\mathbb{C})$ – at low momentum. So, the first-order nonlinear ordinary differential equation for $D_H(\mathbb{C})$ thus were

$$\frac{1}{2 - D_H(\mathbb{C})} = 1 + \ln(2)\,[\,D_H'(\mathbb{C})\,]^2 \,, \qquad \text{Eq.(1)}$$

the idea of using Hausdorff-dim's derivative going back to A. Douady's and G. Havard & M. Zinsmeister's results that the derivative of the Hausdorff dimension tends to infinity for certain values of $\mathbb{C}$ (e.g. when $\mathbb{C} \rightarrow 1/4$, along the Mandelbrot set's real c-axis). Implicit solutions of Eq.(1) were calculated via Wolfram.*Mathematica* of the excellent Wolfram| Alpha™ Computational Knowledge Engine (www.wolframalpha.com) and are

$$c_1 \pm \frac{\mathbb{C}}{\sqrt{\ln(2)}} = -\sqrt{(1 - D_H(\mathbb{C}))}\,\sqrt{(D_H(\mathbb{C}) - 2)} - \mathrm{ATAN}\left(\sqrt{(D_H(\mathbb{C}) - 2)} \,/\, \sqrt{(1 - D_H(\mathbb{C}))}\right), \qquad \text{Eq.(2)}$$

the (+) solution giving the $D_H'(\mathbb{C}) \geq 0$ condition for particle acceleration right. The constant is fixed via the $D_H(J_0) = D_H(0) = 1$ (rest frame) initial value.