Appendix(B) concerning special Julia sets' Hausdorff dimension

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Abstract
The appendix B contains (our) considerations on a possible role of (quadratic) Julia sets' Hausdorff dimension in particle dynamics. A likely link to special relativity, at the moment in the classical dynamics limit only, is presented.

Contents
Referring to our preceding statement that the fractal dimension is thought to be a sort of dimensional eigenvalue analogue to non-bonded quantum states' continuous energy (so http://culetto.at/private_research_associates/sciencephilosophy9.pdf), indications for a link of this kind shall be shown. In a scenario where nature indeed uses Mandelbrot's set M as (a) control space, expressing M's combinatorial features via its adjoined Julia sets' dynamics – M and $J_c(z)$ belonging to the iterative $z \rightarrow z^2 + c$ map, with parameter $c \in \mathbb{C}$ and variable $z \in \mathbb{C} \cup \{\infty\}$ – $J_c(z)$'s properties are expected to manifest themselves in an observable manner. In case of small complex λ, the Hausdorff dimension of the Julia set J belonging to the iterative $z \rightarrow z^q + \lambda$ map was calculated by D. Ruelle (1982, Ergod.Th. & Dyn. Syst. 2, 99 -107), and, converted to our notation ($t = D_H(J_c)$, $q = 2$ and $\lambda = c$), is

$$D_H(J_c) = 1 + \frac{|c|^2}{4\ln(2)} + \text{ higher order terms in } c\,, \qquad \text{Eq.(1)}$$

Eq.1 again to be rewritten in order to avoid confusion with relativistic terminology (c there denoting the speed of light, so the parameter c from now on represented by $\mathbb{c}$), yielding

$$D_H(J_{\mathbb{c}}) = 1 + \frac{|\mathbb{c}|^2}{4\ln(2)} + \text{ higher order terms in } \mathbb{c}. \qquad \text{Eq.(2)}$$

S. Haas' source comment (2003, thesis/Harvey Mudd College) summarizes that Ruelle proves the continuity of the Hausdorff dimension (thus $D_H(J_{\mathbb{c}})$ is a continuous function of the parameter $\mathbb{c} \in \mathbb{C}$) for hyperbolic components of the connectedness locus of rational functions. Eq.2's shape coincides with the Taylor's-series expansion of Lorentz-Einstein's equation for the relativistic mass dependence on particle velocity v in case of $v/c = \beta \ll 1$:

$$\frac{m}{m_o} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \text{ higher order-in-}\beta\text{ terms}. \qquad \text{Eq.(3)}$$

In order to reproduce the correct classical kinetic energy of mass m_o, $D_H(J_{\mathbb{c}})$ expression's 2nd term on the right-hand side had to be the same as Eq.3's corresponding one, i.e.

$$\beta_D = \frac{|\mathbb{c}|}{\sqrt{2\ln(2)}}\,, \qquad \text{Eq.(4)}$$

which – if there indeed existed such $D_H(J_\mathbb{C}) \leftrightarrow$ relative energy E/E_o relation – be named "Dorfer's β" by us (in memory of Harald Dorfer, a friend and a special relativity freak who was killed in a motorbike crash in student days). The relation (if such, and if for dynamical then for gravitating mass too) holds up to αc and much more in v, α being the fine-structure constant, and so well reaches the quantum domain. At least for the classical mechanics' domain, $D_H(J_\mathbb{C})$ or its $\mathcal{O}(|\mathbb{C}|^2)$ term could be some series expansion's leading term(s).

Finally, for the relativistic domain, relative energy E/E_o likely becomes a function of $J_\mathbb{C}$'s Hausdorff dimension (fulfilling the continuity demand). As all of the fractals involved are subsets of the plane, their Hausdorff dimension must stay less (or equal) 2. Informally, when the fractal eventually "fills" the plane, there is no space left for (quantum) particle's further acceleration, i.e. neither for mass- nor for velocity-increase (except for special / extremely rare situations, likely linked to complicated object connectivity).The empirical existence of a *speed of action's propagation* thus went back to geometrical restrictions, another indication that one couldn't escape geometrization. The limiting Hausdorff-dim 2 of the fractals in relativistic dynamics likely matched the $\approx M_P c$ (or the infinite?) momentum limit, M_P being the Planck mass. As Feigenbaum's number for an area-preserving 2-dimensional map, as well as the external angles belonging to $\mathbb{C} = -2$, Mandelbrot set's left end and to the Myrberg-Feigenbaum point $\mathbb{C}_D$ (and both of these $\mathbb{C}$ values $\in \partial M$) appeared in scaling of the Planck mass – electron pair rest-mass ratio (for details see our 2006 paper http://culetto.at/private_research_associates/sciencephilosophy.pdf), a subset/region of M's boundary might be providing appropriate Hausdorff-dimension-2 Julia sets (see also M. Shishikura, arXiv:math/9201282v1 (1991) and Ann. Math.**147**, (1998), 225-267). Or at least not too pathological ones with $D_H(J_\mathbb{C}) = 2 - \varepsilon$, $\varepsilon > 0$.

Concluding, one should be prepared to face formidable problems: At the boundary of the connectedness locus, as *connected objects* only are players in the conventional game, but maybe just *connected-on-average* ones due to some digital "geometrical uncertainty relation" (from rapid fluctuations in $\mathbb{C}$) permitted. One might have to split the problem in a way that quantum particle properties as "very internal structure of the particle trajectory" (in A. Bhattacharya et al.'s diction, arXiv:1109.3543v1 [astro-ph.CO], 2011) be treated on a smaller scale, using $J_\mathbb{C}$ mediators with $\mathbb{C}$ from corresponding parts in an ε-stripe along ∂M (the fluctuation region), and particle motion on a larger scale via such starting from J_0 with D_H=1 in the rest frame. In the small β_D region already, the $J_\mathbb{C}$s involved are extended things and "point particles" in one, by their fractal outline and the inner attractor. In simple terms, the fractal outline (a fractally deformed circle) there gets more and more crumpled as the (quantum) particle momentum increases. As far as we could find out, there is no (published) algebraic expression yet of $D_H(J_\mathbb{C})$ for $\mathbb{C}$ from near to 2 likely necessary for the $f(D_H(J_\mathbb{C}))$'s formulation. The whole problem's degree of sophistication very much depends on the question, whether almost all of the $\mathbb{C}$ values for $f(D_H(J_\mathbb{C}))$ can be restricted to M's big cardioid. As (integer) charge quantization apparently goes along with period-2^0 oscillations (see our file cited above), there is hope that the answer could be affirmative. And the relative energy E/E_o's continuity, experimentally proven to up into the ultrarelativistic region, likely forces continuous $f(D_H(J_\mathbb{C}))$ and $D_H(J_\mathbb{C})$ too, if such $E/E_o = f(D_H(J_\mathbb{C}))$ relation. But the prospect is dim if nature had decided in favour of a "primary bifurcations"- scenario, or even to walk "Shishikura's path" at ultra-high energies, via "secondary bifurcations" and Julia sets drastically inflating in parts which themselves can have Hausdorff-dim close to 2. Perturbation of the underlying iterative maps themselves may indeed occur in the extreme.