Appendix concerning special Mandelbrot set features

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Abstract
The appendix contains approximate functions/expressions in the Mandelbrot set context, concerning external angles ξ(c), bifurcation parameters (c_k) and other real c-axis values.

Keywords: Mandelbrot set, external angles, bifurcation parameters, main bifurcation series, approximations

Contents
For visualization and rough considerations using external angles ξ(c) accessory to real c values $\in [-2, c_D]$ as well as to the main bifurcation series on the real c-axis segment $[c_D, 1/4]$ of the Mandelbrot set, $M = \{c \in \mathbb{C}$: Julia set J_c for given c is connected$\}$, holomorphic functions approximating the discrete ξ values were looked for. Fit precision to some 10^{-3} and elementary functions proved to be sufficient for the task. The unusual terminology of angles goes back to calculations using external angles in the electromagnetic coupling α (fine-structure constant) context, so the usual α(c) spelling for angles was abandoned.

In a relatively straightforward procedure, remarkably simple expressions for continuation of the discrete $\xi(c_k)$ and ξ(c), for both lower and upper external angles each, were found:

$$\xi(c) \approx 1 + \frac{2P}{\pi} \mathrm{ATAN}\left(\frac{2(c - 1/4)}{(c - c_D)}\right) \quad \text{and} \quad - \frac{2P}{\pi} \mathrm{ATAN}\left(\frac{2(c - 1/4)}{(c - c_D)}\right) \quad \text{for } c \in [c_D, 1/4],$$

$$\xi(c) \approx \frac{1}{2} \pm \left(\frac{1}{2} - P\right)\left(1 - \frac{(c - c_D)^2}{(2 + c_D)^2}\right)^{1/2} \quad \text{for } c \in [-2, c_D],$$

where P is the Thue-Morse constant, c_D the Myrberg-Feigenbaum point's coordinate, and c are real c-axis values of M. For external angles ξ are ≥ 0 and counted modulo 1, the fit functions' range of applicability is correspondingly restricted.

Our heuristic study, tracing a possible role of fractal geometry in scaling electrodynamics' fundamentals, suggested some relations between the $\xi(c_k)$ of the main bifurcation series and specific charge or between ξ(c), $c < c_D$ and specific (rest) mass, respectively. Indeed, the $\xi(c)\xi(c_k)$ product appeared in correction terms (maybe from charge-mass interaction). The continuous ξ(c) fit curves' significance (if any) or such of better approximations is still unknown, but the $(c - c_D)^2/(2 + c_D)^2 + (\xi(c) - 1/2)^2/(1/2 - P)^2 = 1$ ellipse would reach until $(2 + 2c_D)$ in c, i.e. between the first and second bifurcation of the main series. Thus, one maybe could expect an observable effect in quark substructure of our hypothetical view. Furthermore, the $\Delta\xi = (\xi(-2) - \xi(c_D)) = (1/2 - P)$ difference in external angle, if significant in this connection, additionally appeared between the "analytically continued" $\xi(c)$ and the approximated, continuous $\xi(c_k)$ curve at $c_{\Delta\xi} \approx -0.92$. The figure below shows discrete ξ_u (upper external angle) values belonging to the main bifurcation series incl. the Myrberg-Feigenbaum point, to the Großmann-Thomae band merging point, to c_1 of the secondary Mandelbrot set and to the left end of M (data points), approximated by fit functions $\xi_u(c)$.

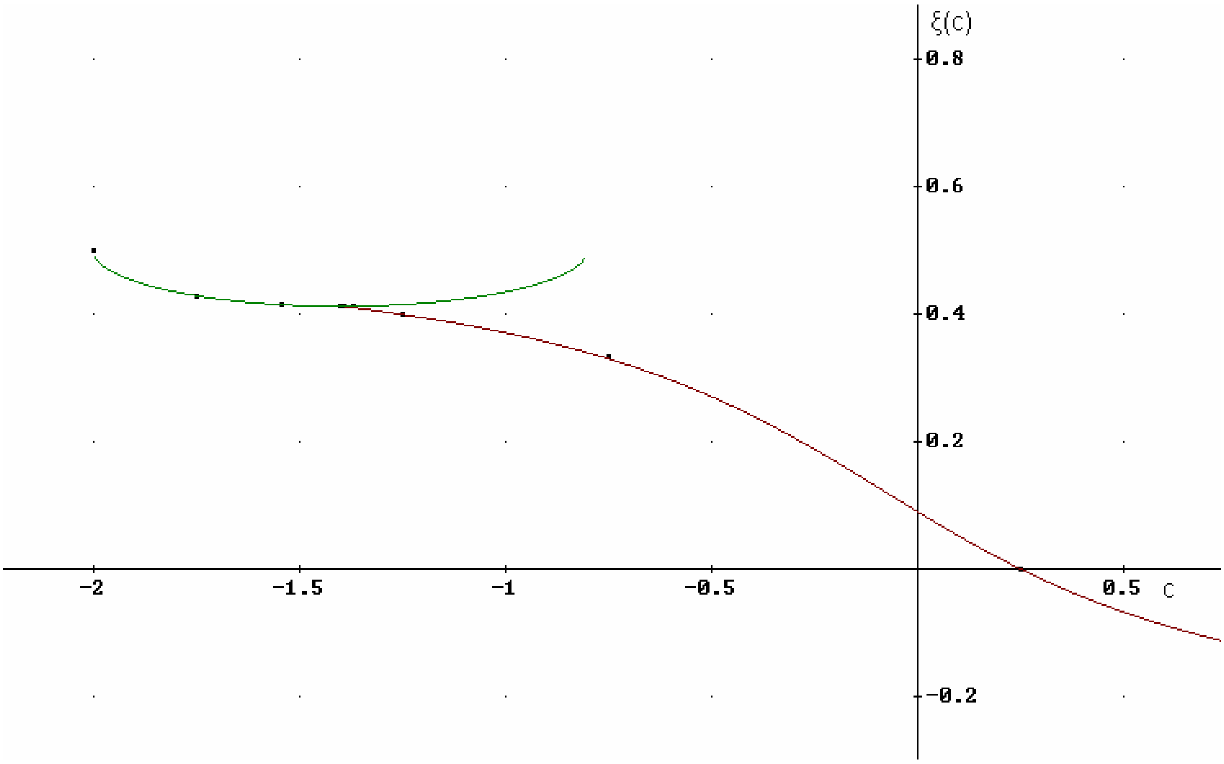

Figure: Discrete upper external angles and corresponding fit functions $\xi_u(c)$. For details see the listing above; data points mentioned are arranged from right to the left.

Concluding, one gets the impression that the possible bond of external angles to physical observables (if the corresponding working hypotheses turned out to be true) goes back to a situation with *phases' extraordinary relevance* (well beyond their mere presence in non-integrable phase factors, as could be the case for some sophisticated phase modulation or/and coding at constant analogue (or even digital) amplitude, where phase functionals likely carry the entire meaningful information), which seems to escape proper treatment within the conventional gauge theoretical framework.