M-set hyperbolic components tuning the matter / dark matter / dark energy pie chart? Details - Internal Comments (Appendix, March 2019)

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There are a few further details to be added to our said internal comments on the matter, which is presented on our www.culetto.at website. The terminology remains unchanged: M-set big cardioid's area $A_{2^0} = 3\pi/8$, the "inflated" cardioid's area $A_{2^0 \mathrm{infl}} = 3\pi(1/4 + c_D)^2/2$ (c_D is the main bifurcation series Myrberg-Feigenbaum point's coordinate $-1.401\,155... + 0i$), the maximally permissible c-disc $A_{c\text{-disc}} = 16\pi$ for preservation of M's combinatorial features, as well as the z = 0 cosmological parameters $\Omega_{m,0}$ and $\Omega_{dm,0}$ being compared with the Planck collaboration's 2015 results.

$$\Omega_{m,0} = \frac{2(A_{2^0})}{A_{c\text{-disc}} - 2A_{2^0}} = 0.049\underline{1}80... \qquad \text{Eq.(1)}$$

$$\Omega_{dm,0} = \frac{2(A_{2^0 \mathrm{infl}} - A_{2^0})}{A_{c\text{-disc}} - 2A_{2^0 \mathrm{infl}}} = 0.268\underline{2}41... \qquad \text{Eq.(2)}$$

The inflated cardioid area generating circles' diameter $a_{2^0 \mathrm{infl}} = |c_D| - 1/4$ was found by extrapolating that of the main cardioid ($a_{2^0} = |c_{k=2}| - 1/4$; $c_{k=2} = -3/4$) to the inf-k-limit. In order to possibly generate any statements about the non-zero redshift cosmological parameters in case of Ω_{dm}/Ω_m being approximately $\Omega_{dm,0}/\Omega_{m,0}$, generalization of the Eqs. (1) and (2) by use of Mandelbrot set real c-axis $|c|$ values as *common variable* is thought of being a viable route. The cardioids' areas ($3\pi a_i^2/2$) inserted, these above two relations yield

$$\Omega_m(|c|) = \frac{3|c|^2}{16 - 3|c|^2} \quad \text{, giving } \Omega_{m,0} \text{ as reference for } |c| = 1/2 \text{ , and} \qquad \text{Eq.(3)}$$

$$\Omega_{dm}(|c|) = \frac{3f(|c|)^2 - 3|c|^2}{16 - 3f(|c|)^2} \quad \text{, which should yield } \Omega_{dm,0} \text{ for } |c| = 1/2. \qquad \text{Eq.(4)}$$

As $f(|c|)$ then has to be $(|c_D| - 1/4)$, the ansatz of $f(|c|) \;\alpha\; \mathrm{ATAN}((|c_D| - 1/4)|c|)$ would be chosen, and the proportionality factor c_0 get calculated from the $\Omega_{dm}/\Omega_m \approx \Omega_{dm,0}/\Omega_{m,0}$

constraint. The ansatz was motivated by an older result of ours in approximation of the Mandelbrot set main bifurcation-series root c_k's discrete upper external angles then by a continuous fit function of ~ ATAN((g(c)) shape for $c \in [c_D,1/4]$. By using expressions and constants commonly found in 1-2D nonlinear dynamics, the factors $c_o = (2 + P/2)$, P being the Thue-Morse constant (= upper external angle of c_D in the M-set), also $c_o = \pi(|c_D| \ln(\delta_{2D})/\ln(\delta))^{1/2}/2$ (δ and δ_{2D} being the Feigenbaum numbers for a 1D or an area-preserving 2D map, respectively) are gotten as pretty good numerical fit results. Thus, the following approximation curve,

$$\Omega_{dm}(|c|) \approx \frac{3[c_o \mathrm{ATAN}((|c_D| - 1/4)|c|)]^2 - 3|c|^2}{16 - 3[c_o \mathrm{ATAN}((|c_D| - 1/4)|c|]^2} , \qquad \text{Eq.(5)}$$

is producing $\Omega_{dm}(|c|)$ results respecting the $\Omega_{dm}/\Omega_m \approx$ const.- constraint, at least up to $|c| = 1/2$ when using the more complicated c_0, but with some deviations tolerated then further up to $|c| \approx 0.68$ where one encounters already ca. twice the $\Omega_{m+dm,0}$ value. The formal constraint equation, which only makes sense in the treated |c|-region or maybe even slightly less, is

$$F(|c|) = \mathrm{SQRT}[(16|c|^2(1 + \Omega_{dm,0}/\Omega_{m,0}) - 3|c|^4)/(16 - 3|c|^2(1 - \Omega_{dm,0}/\Omega_{m,0})]. \qquad \text{Eq.(6)}$$

In a FLRW-universe context, the $\Omega_{m+dm}(a)$ parameter – a here denoting the scale factor a(t) versus a_i for cardioids – in the case of flat space & still negligible $\Omega_{radiation}$ would be

$$\Omega_{m+dm}(a) \approx \frac{\Omega_{m+dm,0}}{a + \Omega_{m+dm,0}(1 - a) + \Omega_{\Lambda,0}(a^3 - a)} , \qquad \text{Eq.(7)}$$

and our related expression is

$$\Omega_{m+dm}(|c|) \approx \frac{3|c|^2}{16 - 3|c|^2} + \frac{3[c_o \mathrm{ATAN}((|c_D| - 1/4)|c|)]^2 - 3|c|^2}{16 - 3[c_o \mathrm{ATAN}((|c_D| - 1/4)|c|]^2} . \qquad \text{Eq.(8)}$$

When substituting the variable |c| by the derivative of ATAN(a), i.e.$1/(1 + a^2)$ in Eq.(8), c_o being $\pi(|c_D| \ln(\delta_{2D})/\ln(\delta))^{1/2}/2$, pretty good fit to the $\Omega_{m+dm}(a)$ curve is obtainable, at least around a = 1 (where |c| = 1/2). For a < 1, and if staying in the region where the relation makes sense, the fit curve would show a little less thinning out of matter and dark matter compared to Eq.(7), whereas in (our) universe's future, dilution of the said components would proceed somewhat faster.

Under the same conditions where Eq.(7) holds, $\Omega_{\Lambda}(a)$ in the FLRW universe would be

$$\Omega_{\Lambda}(a) \approx \frac{\Omega_{\Lambda,0}\, a^3}{a + \Omega_{m+dm,0}(1 - a) + \Omega_{\Lambda,0}(a^3 - a)} \quad , \qquad \text{Eq.(9)}$$

and our related expression for $\Omega_{\Lambda}(|c|)$ (from a $\Omega_{\Lambda,0}$ approximant found accidentally) is

$$\Omega_{\Lambda}(|c|) \approx 1 - \frac{3[(4 + c_D)|c|]^2}{16} \quad . \qquad \text{Eq.(10)}$$

When substituting the variable $|c|$ by $1/(1 + a^2)$ in Eq.(10), pretty good fit to the $\Omega_{\Lambda}(a)$ curve Eq. (9) is obtainable this time, at least around $a = 1$ (where $|c|=1/2$). Eq.(10) is a "continuation" of our $\Omega_{\Lambda,0}$ approximation which was $\Omega_{\Lambda,0} \approx (A_{c\text{-disc}} - 3\pi(4 + c_D)^2/4)/A_{c\text{-disc}}$, this yielding $(1 - 3(4 + c_D)^2/64$. The subtracted term in the brackets of the former formula is *half the area* of a cardioid with generating circle diameter $(4 + c_D)$. As *double* covering of cardioid areas in the Ω_m and Ω_{dm} context was a kind of accounting for fermion spin (this inspired by Yuval Ne'eman's then discussion statement on the appearance of the double covering of the Lorentz group in: To fulfill a vision, Y. Ne'eman (ed.), 20 (1981), Addison-Wesley), one may ask what *half* covering could mean in connection with Ω_{Λ}, thought of being the $\Omega_{\text{dark energy}}$ part of Ω_{total}. Restriction to single covering of the cardioid area's half located in the upper complex c-parameter half-plane would produce the right result, but why do so? In the present-time dark matter and dark energy share context, the involved cardioids' generating circle diameters are $\mathrm{ABS}(1/4 + c_D)$ and $\mathrm{ABS}(4 + c_D)$, these expressions maybe co-resulting from a duality. But there is an important caveat:

Such considerations would only make sense if not being pursuing a just coincidental relation, or if not being caught in a kind of self-trapping circular arguing, respectively. From our older results we probably may conclude that the Mandelbrot set, if used as a control space by nature, would primarily be effective in shaping the asymptotic behavior of *relative*, dimensionless physical quantities. The M-set, being a subset of a complex parameter *plane*, would likely just support *locality* in case of a Riemann-curved space, but it should apply to *extended scales* in the flattened-out-space approximation case.