Fractionally charged massive (FCM) leptons predictions: Summary

(F.J. Culetto and W. Culetto, Private Research-Associates, dated: February 2018)

Appendix 5 concerning fractionally charged massive lepton $m_{X,Y}$ - estimation

(Private Research-Associates*), dated: Nov. 2015)

As had already been communicated in the previous summaries/appendices on the said estimation procedure, there are a few degrees of freedom in eventually arriving at some final result. Taking the last appearance period-5 cardioid (on M's real c-axis) cusp's lower external angle 16/31 as the analogue to 1/2 in case of the electron, the fractional charge quantum numbers and the estimated $m_{X,Y}$ – values are:

Case (1): Relation only making use of Mandelbrot set's main bifurcation series' Myrberg-Feigenbaum point's *upper* external angle P (= the Thue-Morse constant) yields particles

(– 3/7, + 4/7), 5.006 686 [TeV/c^2] each, up to some small electromagnetic self-energy difference; scaling the error bars of the initial $m_{\mu,\tau}$ - fit, these and the other results' standard deviations are likely to be some 0.0001 [TeV]

Case (2): Relation making use of *both* of the main series Myrberg-Feigenbaum point c_D's external angles, i.e. P *and* (1 – P), yields

(– 3/7), 5.006 686 [TeV/c^2]
(+ 4/7), 5.975 628 [TeV/c^2]

Case (3): Relation making use of the *secondary Mandelbrot set's* main bifurcation series' Myrberg-Feigenbaum point ${}_2c_D$ = – 1.779 818… and of its (approximated) external angles ≈ (1828/4097, 2269/4097) yields

(– 3/7), 6.614 654 [TeV/c^2]
(+ 4/7), 7.369 470 [TeV/c^2]

There is some chance left that the communicated particle masses' values could have been upper bounds. If one took the said hyperbolic p5 component (antenna) tip's *lower* external angle – i.e. used G. Pastor/M. Romera/G. Álvarez/F. Montoya's *order* ∞ *Fourier harmonic* $H_F^{(\infty)}$(15/31, 16/31), which yields a lower external angle of 511/992 – instead of the cardioid cusp's 16/31 which entered the fit formulae via ξ_{0i} = (1 + 16/31), the masses would be:

Case (1): (– 3/7, + 4/7), 2.325 585 [TeV/c^2] each, up to a small em self-energy difference

Case (2): (– 3/7), 2.325 585 [TeV/c^2]
(+ 4/7), 2.775 654 [TeV/c^2]

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Case (3): (– 3/7), 3.072478 [TeV/c^2]
(+ 4/7), 3.423088 [TeV/c^2],

what would notably enhance the probability of being detectable in the LHC-experiments (at least in the cases (1) and (2)), both from a cross section and statistics point of view.

Erratum: Unfortunately, an incorrect value for ${}_2c_D$, the secondary Mandelbrot set main bifurcation series' Myrberg-Feigenbaum point got stored and was subsequently used. As the said point coordinate's absolute value just appears as a pre-factor in the approximate equation, the error concerning all of the older case (3) particle mass results can be easily removed by multiplying each of these by the ratio (1.77981807/1.78644025).

Appendix 4 concerning fractionally charged massive lepton $m_{X,Y}$- estimation
(Private Research-Associates*), dated: May 2015)

As had already been communicated in the previous summaries/appendices on the said estimation procedure, there are a few degrees of freedom in eventually arriving at some final result. In the (integer) charged leptons' family context the absolute error in $m_{\mu,\tau}$ was < $\mathcal{O}$(0.1MeV) using the original mass-fit relations. Unfortunately, no reliable /well-argued error bar can be given for the relations extrapolated to external angles >1/2 and such plus one full turn, so the numeric results are formally reproduced just down to the MeV-range. The massive (hypothesized) leptons (if such) come with (–3/7, +4/7) electric charge QN – with a very small probability of finding charges' sign reversal realized – and, from their charges' link to a Mandelbrot set M's special lower and upper external angle, could so be named up/down - xxxxxx [1)], analogous to quark-nomenclature. Particle - antiparticle pair production energy thresholds are shown *for orientation purposes*, but measurable *mass asymmetry* (→ deviation from $2m_{X,Y}$) with increasing oscillation-period k is pretty likely: for the particles treated this period is 3, whereas k = 2 for quarks and 1 for electron/muon/tau.

Case (1): Relation only making use of Mandelbrot set's main bifurcation series' Myrberg-Feigenbaum point's *upper* external angle P (= the Thue-Morse constant) yields particles

(– 3/7, + 4/7), 5.006686 [TeV] each; (– 3/7, + 3/7), (– 4/7, + 4/7), 10.013372 [TeV]

Case (2): Relation making use of *both* of the main series Myrberg-Feigenbaum point c_D's external angles, i.e. P *and* (1 – P), yields

(– 3/7),	5.006686 [TeV];	(– 3/7, + 3/7),	10.013372 [TeV]
(+ 4/7),	5.975628 [TeV];	(– 4/7, + 4/7),	11.951256 [TeV]

1) Having a **fitting working title** for the said hypothesized *exotic leptons* would make sense. From a purely technical point of view – their fractional charge quantum no.'s being linked to so-called external arguments, often called external angles too – "*up/down exargon*" or "*up/down exanglon*" (preferred by conspiracy-theorists) would maybe apply. Honouring Adrien Douady's† dictum that there seem to be indications "*that external arguments are not just a mathematician's trick, a useful artefact, but that they really occur "in nature*"", the particles may be named "*up/down douadon*" or *douadyon*, respectively. Beware of naming the particles after us, because "*culon*" would mean a *(big)ass-particle*, and the *–on* added to our full name then stand for the *small-ass-version* of it!

Case (3): Relation making use of the *secondary Mandelbrot set's* main bifurcation series' Myrberg-Feigenbaum point ${}_2c_D = -1.786\,440\ldots$ and of its (approximated) external angles ≈ (1828/4097, 2269/4097) yields

(– 3/7),	6.639 265 [TeV];	(– 3/7, + 3/7),	13.278 530 [TeV]
(+ 4/7),	7.396 890 [TeV];	(– 4/7, + 4/7),	14.793 780 [TeV]

Concluding, the cases (1) and (2) are pretty much more likely to be possibly realized, from both, the available LHC energies range and our mass estimation's philosophy/combinatorics.

Appendix 3 concerning fractionally charged massive lepton $m_{X,Y}$ - estimation

(Private Research-Associates*), dated: April 2015)

There are some details to be added to our summaries "Appendix concerning fractionally charged massive lepton $m_{X,Y}$ - estimation", dated: Jan. & Feb. 2015. As would be described there, the following set of formulae was used by taking the expressions' right hand sides appropriately to yield $2(M_P/2m_e)(m_i/M_P)(m_e)$ [TeV], where m_e is the CODATA 2010 electron rest mass value measured in TeV units. All of the symbols used before remain unchanged:

$$\frac{M_P}{2m_e} \approx \frac{\sqrt{2}\,\ln(\delta_{2D})}{\sqrt{\pi P}|c_D|\ln(\delta)} \exp(\gamma^{1/2} e^{\pi/2+1/2} \pi^{e/2+1/2}) \qquad \text{Eq.(1)}$$

$$\frac{M_P}{m_i} \approx \frac{2\ln(\delta_{2D})}{\sqrt{\xi_{0i} P}\,\Gamma(\xi_{0i})|c_D|\ln(\delta)} \exp(\gamma^{1/2} e^{B(1/2,\,\xi i)/2+1/2} B(1/2,\,\xi_i)^{e/2+1/2}) \qquad \text{Eq.(2)}$$

$$\xi_i = (1/2 + \frac{\pi^2\Gamma(\pi/2+1/2)^2\xi_{0i}^{\,2}}{4\Gamma((e/2+1/2)\ln(\pi))\Gamma(1/2-\xi_{0i})}) \qquad \text{Eq.(3)}$$

Eq.(3) had been found in a tentative procedure, when testing various formulae nonlinear in ξ_{0i} with a decreasing degree of self-similarity compared to parts of Eq.(1). For $\xi_{0e} = 1/2$, the generalized Eq.(2) reduces to Eq.(1). From the $1/\sqrt{\pi P/2}$ pre-factor, its geometric mean structure would then be used again in the charged leptons' family context. As most of the argumentation relies on *Mandelbrot set's* (combinatorial) features, the just small residues of self-similarity eventually left in Eq.(3) didn't come as a surprise. But the maybe reason behind could possibly get such: our meanwhile found empirical relation between massive gauge boson masses and the $\Gamma(\pi/2+1/2)^2$ and $\Gamma((e/2+1/2)\ln(\pi))$ factors of Eq.(3). With $m(Z^0) = 91.1876$ [GeV], $m(W^\pm) = 80.385$ [GeV] the relation $\sqrt{m(Z^0)/m(W^\pm)} \approx \Gamma(\pi/2+1/2)^2$ is almost perfect, the left-hand side yielding 1.065 075 versus 1.065 07$\underline{8}$8… on the right.

An even better fit in terms of the vector boson masses and the arithmetic - geometric mean M(1/2, P), i.e. such of the external angle values belonging to the Mandelbrot set's left end and the main series' Myrberg-Feigenbaum point c_D would be found:

$$\big(M(1/2, P)m(Z^0) + (1 - M(1/2, P))m(W^{\pm})\big)/m(W^{\pm}) \approx \Gamma((e/2 + 1/2)\ln(\pi)) \ , \qquad \text{Eq.(4)}$$

the relation's left-hand side yielding 1.061 168 9 versus Γ's 1.061 169 174… on the right, and $M(a,b) = 0.5\pi / \mathrm{INT}(1/\sqrt{a^2\cos(\Theta)^2 + b^2\sin(\Theta)^2}, \Theta, 0, \pi/2)$ used. Inserting the gamma functions' "equivalents" into Eq.(3) eventually ends up with having the geometric mean of the vector boson masses in the equation's quadratic term's numerator:

$$\xi_i = \left(1/2 + \frac{\pi^2 \sqrt{m(Z^0)m(W^{\pm})}\,\xi_{0i}^2}{4\big(M(1/2, P)m(Z^0) + (1 - M(1/2, P))m(W^{\pm})\big)\Gamma(1/2 - \xi_{0i})}\right) \qquad \text{Eq.(5)}$$

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Appendix 2 concerning fractionally charged massive lepton $m_{X,Y}$ - estimation

(Private Research-Associates*), dated: Feb. 2015)

There are some details to be added to our summary "Appendix concerning fractionally charged massive lepton $m_{X,Y}$ - estimation", dated: Jan. 2015. As would be described there, the following set of formulae was used by taking the expressions' right hand sides to yield $2(M_P/2m_e)(m_i/M_P)(m_e)$ [TeV], where m_e is the CODATA electron rest mass value. All of the symbols used remain unchanged:

$$\frac{M_P}{2m_e} \approx \frac{\sqrt{2}\,\ln(\delta_{2D})}{\sqrt{\pi P}|c_D|\ln(\delta)} \exp(\gamma^{1/2} e^{\pi/2+1/2} \pi^{e/2+1/2}) \qquad \text{Eq.(1)}$$

$$\frac{M_P}{m_i} \approx \frac{2\ln(\delta_{2D})}{\sqrt{\xi_{0i}P}\,\Gamma(\xi_{0i})|c_D|\ln(\delta)} \exp(\gamma^{1/2} e^{B(1/2,\, \xi_i)/2+1/2} B(1/2, \xi_i)^{e/2+1/2}) \qquad \text{Eq.(2)}$$

$$\xi_i = \left(1/2 + \frac{\pi^2\Gamma(\pi/2 + 1/2)^2 \xi_{0i}^2}{4\Gamma((e/2 + 1/2)\ln(\pi))\Gamma(1/2 - \xi_{0i})}\right) \qquad \text{Eq.(3)}$$

According to the argumentation already given – extending Eq.(3) to continuous external angle values $\xi_0 \in [P, 1/2]$, and further "analytical continuation" to ξ_0 values >1/2 and higher winding number – one ends up with relations containing the $\xi_{0i} = (1 + 16/31)$, one plus the angle accessory of the leftmost accessible chaotic c-value, coming along from the *secondary M* main series' start at c = – 1.75. ${}_2$M periodic c-region's 1st bi-accessible real c-axis point's external angles (3/7, 4/7) would then be the absolute values of the new leptons' fractional (el.)charge quantum numbers, fractional relative charge linked to upper external angle 3/7 counted negative (from a sign($\xi(c_k)$ – 1/2) term in our corresponding formula for charges linked to external rays in the upper c - halfplane). As the ${}_2$M-cardioid is a period-3 hyperbolic component, a $(-1)^k$ - factor not seen in the 2k-case, could possibly reverse polarity from (– 3/7, 4/7) to (3/7, – 4/7), but pretty likely the more general assumption is going to hold. If the *main-series Myrberg-Feigenbaum point* c_D's external angles (P, 1 – P) *both* had to be used in Eq.(2), the prediction would be (– 3/7; 5.007TeV) and (4/7; 5.976TeV). If instead the *secondary M main series' Feigenbaum point* at c = – 1.786440252 with its approximated external angles (1828/4097, 2269/4097) had to be used, the corresponding results for the (hypothesized) fractionally charged heavy X,Y- leptons eventually would be (– 3/7; 6.639TeV) and (4/7; 7.396TeV). As $m_{X,Y}$ heavily depend on $(1 + \xi_0^{X,Y})$, the very details should not be trusted. No statement on the spin connection is feasible from main and secondary M's combinatorial features (except for the fact that the mass-fit relations as searched for apply to family1,2,3's spin ½ charged leptons), but Eq.(3) extrapolated to higher winding number might possibly point towards a possibility of having spin 3/2 states. If the predicted particles indeed are real, they should be named ± douadyons (after Adrien Douady's† dictum that there seem to be indications "that external arguments are not just a mathematician's trick, a useful artefact, but that they really occur "in nature""). Usage of M- and midget-M sets' (invariant) combinatorial features as a short cut method, reminding of dimensional analysis somehow, could grant a new approach if nature really works that way.

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Appendix concerning fractionally charged massive lepton $m_{X,Y}$ - estimation

(Private Research-Associates, dated: Jan. 2015)

Starting from our previously found *trial-and-error fit relations* (for details see the articles of our culetto.at website), looked for in the 2nd and 3rd generation charged leptons context, we would try to estimate the (hypothesized) new leptons' mass using Eqs.(1) - (3), i.e. by taking $2(M_P/2m_e)(m_i/M_P)(m_e)$ [TeV]. The argumentation then goes like this: Eqs.(2) plus (3), the latter initially for discrete external angles (from the chaotic c-region of the Mandelbrot set M's real c-axis slice), 1/2 for the electron, P (=Thue-Morse constant) for the tau particle and the geometric mean $\sqrt{P/2}$ for the muon, gave pretty good fit in case of m_i for i = e, τ but a less precise value for μ. The other symbols used are: the Planck mass M_P, the Euler-Mascheroni constant γ, the main series Myrberg-Feigenbaum point's coordinate c_D, Feigenbaum's universal δ, Feigenbaum's number for an area-preserving 2D-map δ_{2D} (= 8.721…), external angles ξ_{0i}, as a nonlinear function of the latter ξ_i, Gamma function $\Gamma(\xi_{0i})$ too and finally, Euler's Beta function $B(1/2, \xi_i)$. $\sqrt{\pi P/2}$ would get generalized to $\Gamma(\xi_{0i})\sqrt{\xi_{0i}P}$ for Eq.2

$$\frac{M_P}{2m_e} \approx \frac{\sqrt{2}\,\ln(\delta_{2D})}{\sqrt{\pi P}|c_D|\ln(\delta)} \exp(\gamma^{1/2} e^{\pi/2+1/2} \pi^{e/2+1/2}) \qquad \text{Eq.(1)}$$

$$\frac{M_P}{m_i} \approx \frac{2\ln(\delta_{2D})}{\sqrt{\xi_{0i}P}\,\Gamma(\xi_{0i})|c_D|\ln(\delta)} \exp(\gamma^{1/2} e^{B(1/2,\,\xi_i)/2+1/2} B(1/2, \xi_i)^{e/2+1/2}) \qquad \text{Eq.(2)}$$

$$\xi_i = (1/2 + \frac{\pi^2\Gamma(\pi/2 + 1/2)^2\xi_{0i}^2}{4\Gamma((e/2 + 1/2)\ln(\pi))\Gamma(1/2 - \xi_{0i})}) \qquad \text{Eq.(3)}$$

As a first step in generalization, Eq.(3) would be extended to continuous external angle values $\xi_0 \in [P, 1/2]$, further “analytical continuation” of Eq.(3) to ξ_0 values >1/2 and higher winding number (plus one full turn) would follow, ending up with relations containing the $(1 + \xi_0^{X,Y})$, one plus the angle accessory of the leftmost accessible chaotic c-value, coming from the *secondary M* main series’ start. ${}_2$M periodic c-region’s 1^{st} bi-accessible real c-axis point’s external angles (3/7, 4/7) would then be the absolute values of the new leptons’ fractional (el.)charge quantum numbers, fractional relative charge linked to upper external angle 3/7 counted negative (from a $\mathrm{sign}(\xi(c_k) - 1/2)$ term in our corresponding formula). Checked out G. Pastor et al.’s external arguments in the Mandelbrot set antenna – and the known argument of period-3 oscillations as short cut to chaos in mind – as analogue to main M’s left end the period-5 last appearance cardioid (15/31, 16/31) angles and, as stated before the one > 1/2 of these would be taken, i.e. $(1 + \xi_0^{X,Y}) = (1 + 16/31)$ for ξ_{0i} in the set of formulae above. Unfortunately, $m_{X,Y}$ are that strongly depending on $(1 + \xi_0^{X,Y})$ such that (1 + 16/31) would give resonances at 5.0TeV but (1 + 0.5176) such at 14.6TeV. And if ${}_2$M period tripling Feigenbaum-point’s coordinate and lower external angle had to be used instead of c_D and P in Eq.(2), the 5.0TeV $m_{X,Y}$-resonances would get shifted to 7.4TeV. In (not much trusted) detail the mentioned $m_{X,Y}$- shift would be from 5.007TeV to 7.397TeV. Furthermore, there is no guarantee that Eqs.(2, 3) combined indeed are sufficiently precise in the external angle regions been extrapolated to, but the fit in case of m_μ wasn’t that bad. An entirely unexplained feature of the masses-estimation reported is the extremely small mass result for the M-antenna’s period-5 last appearance cardioid cusp’s upper external angle 15/31 gotten for $\xi_{0i} = (1 + 15/31)$ from the above equations and being 1.29×10^{-20} TeV.

As far as the factorization of the fit-formulae Eqs.(1) and (2) is concerned, a pretty similar shape of the proton-electron rest mass ratio approximation and approximate expressions for the Higgs boson mass might point towards deep connections between these, iff true:

$$\frac{m_p}{m_e} \approx \frac{P\ln(\delta_{2D})}{|c_D|\ln(\delta)}\, e^{\pi+1}\pi^{e+1}\,, \qquad \text{Eq.(4)}$$

the fit value gotten when insertion of 106/257 (= the n=4 approximant to P, the upper external angle accessory of the root of M main series’ 4^{th} bifurcation) instead of P done, been $1836.152\underline{4}54$, i.e. a deviation of 0.12ppm seen from the experimental value.

As another numerological curiosity, with the same kind of “divisors” in factorization, two relations reproducing the Higgs boson mass would get constructed, there in case (1) just assuming period doubling oscillations in 1D and 2D near /in the infinite bifurcation limit and the 1D and 2D maps’ Feigenbaum numbers being standard deviations of Gaussian distribution-densities, i.e. σ_{1D}, σ_{2D}, in case (2) again needing an infinite bifurcation limit, but this time the Mandelbrot set main bifurcation series’ one:

$$(1) \quad \frac{M_{Higgs}}{m_e} \approx \frac{1}{\delta\delta_{2D}}\left(\frac{\ln(\delta)}{\ln(\delta_{2D})}\, e^{\pi+1}\pi^{e+1}\right)^2, \qquad \text{Eq.(5)}$$

the fit-relation using the CODATA electron rest mass giving a 125.146 GeV resonance.

Multiplication of Eq.(5)'s right hand side by the $\ln(\delta_{2D})/(|c_D|\ln(\delta))$ factor already seen in the other relations would then yield

$$(2)\quad \frac{M_{Higgs}}{m_e} \approx \frac{1}{\delta\delta_{2D}}\left(\frac{\ln(\delta)}{|c_D|\ln(\delta_{2D})}\right)\left(e^{\pi+1}\pi^{e+1}\right)^2 , \qquad \text{Eq.(6)}$$

Eq.(6) reproducing a resonance at 125.527 GeV, this nearer to the reported mean value. So, future precision measurements (or more precise LHC results) would maybe be able to distinguish between the cases (if these are not just accidental values/relations-based). Last but not least, our (archetype) formula for the infinite distance limit of electrodynamics' effective coupling constant $\alpha(0)$ is comprising factors of the "universal" pattern and reads

$$\alpha(0) \approx \frac{1}{2\pi\delta^2}\left(\exp\left(-\frac{1}{\gamma(e^{\pi+1}\pi^{e+1} - \pi P/2)}\right)\right) , \qquad \text{Eq.(7)}$$

its value of $7.2973525687 \times 10^{-3}$ well within the best determination's confidence interval.