Short note on a generalized α(0) (fine-structure constant) approximation

F.J. Culetto and W. Culetto*)
Private Research Associates, Stallhofen 59-60, A-9821 Obervellach, Austria
*Electronic address: werner.culetto@inode.at
(Dated: November 2, 2007)

Abstract
Extending our previous tentative & heuristic work on the possible role of fractal geometry (understood in a general sense) in scaling electrodynamics' fundamentals, an attempt toward generalization of the found α(0) approximation in order to fit the charged lepton generations context is reported.

Keywords: Mandelbrot set, α(0) approximation, coupling constant, external angle ξ, period doubling, lepton mass spectrum, Planck mass scale, fourfold external angle

Contents
Making direct contact of Mandelbrot set's (1) features like external angles (2), bifurcation roots, ratios of successive bifurcation root distances (e.g. in the infinite bifurcation limit) etc. to physical observables, the α(0) approximation Eq.(1) was found, which suggests a possible role of fractal geometry (understood in a general sense) in electron rest mass fine-tuning and charge quantization (3) and reads

$$\alpha(0) \approx \frac{1}{2\pi\delta^2}\left[\exp\left(-\frac{1}{\gamma(e^{\pi+1}\pi^{e+1}-\pi P/2)}\right)\right], \qquad \text{Eq.(1)}$$

where α(0) is the infinite distance limit of the electromagnetic force's effective coupling constant, δ Feigenbaum's universal number, γ the Euler-Mascheroni constant and P the Thue-Morse constant. In search of the origin of the e↔π dual, large correction term in Eq.(1), $\exp(\gamma^{1/2}e^{\pi/2+1/2}\pi^{e/2+1/2})$ turned out to be of order Planck mass M_P divided by twice the electron rest-mass m_e and finally ended up in the quite precise (to 6ppm) tentative fit of the masses' ratio Eq.(2) by (3), using the M_P and m_e CODATA 2002 values:

$$\frac{M_P}{2m_e} \approx \frac{\sqrt{2}\,\ln(\delta_{2D})}{\sqrt{\pi P}|c_D|\ln(\delta)}\exp(\gamma^{1/2}e^{\pi/2+1/2}\pi^{e/2+1/2}), \qquad \text{Eq.(2)}$$

where c_D is the Myrberg-Feigenbaum point's coordinate and δ_{2D} Feigenbaum's constant for an area-preserving 2-dimensional map (4, 5). Looking for a generalization of Eq.(2) to fit 2nd and 3rd generation charged leptons, the exchange of the e↔π dual expression for $e^{B(1/2,\, f(\xi))/2\,+1/2}\,B(½,\, f(\xi))^{e/2\,+1/2}$ (B being Euler's Beta function) and following the $\sqrt{\pi P/2}$'s reformulation in terms of the Mandelbrot set's real c-axis values and accessory external angles ξ(c) from (3), $\Gamma(\xi(-2))\sqrt{\xi(-2)\xi(c_D)}$, finally led to the more general form of the M_P/m_i approximation, particle index i = e, μ and τ, reported in (6):

$$\frac{M_P}{m_i} \approx \frac{2\ln(\delta_{2D})}{\sqrt{\xi_{oi} P}\,\Gamma(\xi_{oi})|c_D|\ln(\delta)} \exp(\gamma^{1/2} e^{B(1/2,\, \xi i)/2+1/2} B(½, \xi_i)^{e/2+1/2}) \text{ , with} \qquad \text{Eq.(3)}$$

$$\xi_i = (1/2 + \frac{\pi^2\,\Gamma(\pi/2+1/2)^2\,\xi_{oi}^2}{4\Gamma((e/2+1/2)\ln(\pi))\,\Gamma(1/2-\xi_{oi})}) \text{ ,} \qquad \text{Eq.(4)}$$

latter function of external angles ξ_{oi} (ξ_o = 1/2 for the electron, the 2nd and 3rd generation charged leptons' probably corresponding values $\xi_{o\mu} = \sqrt{P/2}$,$\xi_{o\tau}$ = P) likely co-reproducing a nonlinear problem's discrete mass spectrum and containing the exponents of Eq.(2)'s e↔π dual expression, $e^{\pi/2+1/2}e^{(e/2+1/2)\ln(\pi)}$ in exp( )-form, in a slightly self-similar manner.

Rewritten in terms of the M_P/m_e mass ratio approximation Eq.(2), the α(0) approximation Eq.(1) reads

$$\alpha(0) \approx \frac{1}{2\pi\delta^2}[\exp(-\frac{1}{\ln(CM_P^2/m_e^2)^2/4 - \gamma\pi P/2})] \text{ , } \quad C = \frac{\pi P}{8}\frac{c_D^2\ln(\delta)^2}{\ln(\delta_{2D})^2} \text{ ,} \qquad \text{Eq.(5)}$$

the term containing a large mass^2 cutoff (of order Planck mass squared) argued in (7). According to $\sqrt{\pi P/2}$'s reformulation toward a relative variable-mass-at-constant-charge term, the γπP/2 small correction in Eq.(1) generalizes to $\gamma\Gamma(\xi_o)^2\xi_o P$. Changeover I) from Eq.(1)'s e↔π dual large correction term to its e↔B(½, f(ξ_o)) dual analogue, and II) from discrete to continuous ξ_o finally yields $e^{B(1/2,\, f(\xi o))\,+1}B(½, f(\xi_o))^{e+1}$, f($\xi_o$) being ξ of Eq.(4), this extrapolated to continuous external angles. Due to charge quantization and strict conservation of Q_{em}, and α(0) (in cgs-system) being $(\hbar c)^{-1}$ times the elementary charge squared, latter maximally screened by vacuum polarization processes, α(0)'s invariance under mass variation in the charged lepton generations context has to be guaranteed. Lacking kind of a "sum rule" for accomplishing this automatically, use of a trial-and-error fit procedure yielded just a few functions suitable as correction factor to the numerator of α(0) approximation's (Eq.(1)) exponent to render the tested generalized argument quasi-invariable for $\xi_o \in$ [P, 1/2], the denominator's terms' generalization already settled as was described above. At "maximum simplicity", the generalized α(0) approximation reads

$$\alpha(0) \approx \frac{1}{2\pi\delta^2}[\exp(-\frac{\Gamma(4\xi_o)^{2+\pi P}}{\gamma(e^{B(1/2,\, \xi)\,+1}B(½, \xi)^{e+1} - \Gamma(\xi_o)^2\xi_o P)})] \text{ , with} \qquad \text{Eq.(6)}$$

$$\xi = (1/2 + \frac{\pi^2\,\Gamma(\pi/2+1/2)^2\,\xi_o^2}{4\Gamma((e/2+1/2)\ln(\pi))\,\Gamma(1/2-\xi_o)}) \text{ ,} \qquad \text{Eq.(7)}$$

reproducing the α(0)'s CODATA 2002 value almost exactly for external angle 1/2 (i.e. in case of the electron, where Eqs.(6) & (7) reduce to Eq.(1)), but less precise at external angles likely agreeing with the muon (to 816ppb) and the tau (to 14ppb) particle's mass spectral levels. From α(0) approximation's form − still vaguely reminding of a Gaussian distribution density squared − with two quadratic terms in the exponent's denominator (i.e. $[(\gamma^{1/2}e^{B(1/2,\ \xi)/2+1/2}B(½, \xi)^{e/2+1/2})]^2$ and $[\gamma^{1/2}\Gamma(\xi_o)\sqrt{\xi_o P}]^2$), the numerator function found, this being $[(\Gamma(4\xi_o))^{1+\pi P/2}]^2$, fits the situation. Quadrupling of the external angle is a feature not observed before in our tentative calculations and likely is to be viewed in the fermion spin context. After all, interaction terms being smaller than the $\gamma\Gamma(\xi_o)^2\xi_o P$ correction term (which is $\gamma\Gamma(\xi_o)\Gamma(\xi_o+1)P$ in more suitable formulation) could be missing too and thus limit the precision of fit on principle. Additionally, quality of fit already suffers from imperfection of the $\xi(\xi_o)$ (and maybe also $B(1/2, \xi(\xi_o))$) functional dependence compared to an "exact" one, seen in the M_P/m_i, i = e, μ and τ fit context too.

Reformulation of Eq.(6) via division of the exponential function argument's numerator and denominator by the numerator function for comparison with α(0) as given by Eq.(1) yields the denominator's small (and probably charge↔mass internal corrections related) term

$$Z_{int}^2 \equiv \frac{\gamma\Gamma(\xi_o)\Gamma(\xi_o+1)P}{\Gamma(4\xi_o)^{2+\pi P}}\ ,\ \text{with} \qquad \text{Eq.(8)}$$

single, single plus one full turn, or fourfold external angle in Gamma function's argument, the Z_{int}^2 curve accidentally(?) resembling Maxwell's (directionally averaged) distribution.

Concluding, the tentative generalization of the α(0) approximation Eq.(1) in order to fit the charged lepton generations context (Eqs.(6),(7)) seems to be as plausible as incomplete. Of all our trial-and-error fit results, the α(0) approximation Eq.(5), containing the M_P^2/m_e^2 ratio, most precisely reproduces the fine-structure constant's CODATA 2002 value (better than really permissible statistically in its standard deviation context). At best the CODATA 2002 values for α (from electron's experimental and QED gyromagnetic ratio), for M_P and m_e, as well as the used α(0) fit formula maybe coincide with "exact" constants/relations in precision's interval looked at. But values' deviations' mutual compensation is more likely.

References

(1) Mandelbrot, B.B. Fractal aspects of the iteration of z→λz(1-z) for complex λ and z. *Annals New York Acad. Sciences*, 357, 249 – 259 (1980)
(2) Douady, A. Julia Sets and the Mandelbrot Set in: Peitgen, H.-O. & Richter, P.H. The Beauty of Fractals, 161–173 (Springer-Verlag, Berlin-Heidelberg-New York-Tokyo, 1986)
(3) Culetto, F.J. and Culetto, W. Does fractal geometry tune electrodynamics' scales? http://culetto.at/private_research_associates/sciencephilosophy.pdf (2006)
(4) Tabor, M. Chaos and Integrability in Nonlinear Dynamics: An Introduction, 225. (Wiley, New York, 1989)
(5) Weisstein, Eric W. "Feigenbaum Constant". From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/FeigenbaumConstant.html
(6) Culetto, F.J. and Culetto, W. Short note on the particle generations problem. http://culetto.at/private_research_associates/sciencephilosophy2.pdf (2006)
(7) Culetto, F.J. and Culetto, W. Mandelbrot set's links to physics: more of same? http://culetto.at/private_research_associates/sciencephilosophy3.pdf (2007)